1. “How fast do dominoes fall?” Consider an initial configuration of dominoes which
are equally spaced in a straight line. Let \( d \) be the domino spacing, \( h \) the domino
height, \( g \) gravity, \( w \) the domino width, \( t \) the domino thickness and \( V \) the speed
of the wave of falling dominoes, see Figure 1.

(a) Determine a similarity rule for \( V \) by assuming that \( w \) and \( t \) have negligible
affect on the domino motion.

(b) If the ratio \( d/h \) is assumed constant, what is the dependence of \( V \) on \( h \)?

(c) Of \( V \) on \( g \)?

(d) Suppose the effect of \( t \) is not negligible, what happens to \( V \) as \( t \) tends to \( d \)?

2. Hamilton’s Principle for conservative systems states that: The motion of the
system from time \( t_1 \) to time \( t_2 \) is such that the integral

\[
I = \int_{t_1}^{t_2} L dt
\]

where \( L = T - V \) (the Lagrangian), is an extremum for the path of motion.
Suppose that it was known experimentally that a particle fell a given distance \( y_0 \)
in a time \( t_0 = (2y_0/g)^{1/2} \), but the time of fall for distances other than \( y_0 \) were not
known. Suppose further that the Lagrangian for the problem is known, but that
instead of solving the equation of motion for \( y \) as a function of \( t \), it is guessed
that the functional form is

\[
y = at + bt^2.
\]

Suppose the real constants \( a \) and \( b \) are adjusted so that the time to fall \( y_0 \) is
correctly given by \( t_0 \). Use Hamilton’s principle directly to determine \( a \) and \( b \), i.e.,
determine for what values of \( a \) and \( b \) the integral in equation (1) is an extremum.

3. A rod of length \( L \) has its lateral surface perfectly insulated and is so thin that
heat flow in the rod can be regarded as one-dimensional. Suppose that the initial
temperature is given by

\[
T(x, 0) = x(L - x),
\]

and assume that the ends of the rod are insulated, i.e., zero heat flux. Determine
the temperature, \( T(x, t) \), in the rod for \( t > 0 \). Assume that the thermal diffusivity
\( \alpha \) is a constant.
Figure 1: Figure for Problem 1. Falling dominoes.
4. Consider the heat diffusion problem ($\rho = 1$)

\[ c(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left[ k(T) \frac{\partial T}{\partial x} \right]. \]

Assume that the specific heat is of the form $c(T) = c_0 T^r$ and the thermal conductivity is of the form $k(T) = k_0 T^s$.

(a) Show that by setting $U = T^{r+1}$ we can find

\[ \frac{\partial U}{\partial t} = \frac{\partial}{\partial x} \left[ \frac{k_0}{c_0} \frac{U^s}{r+1} \frac{\partial U}{\partial x} \right]. \]

(b) Suppose that $s = 2$ and $r = 2$. Determine the temperature in an infinite rod with the initial condition $T(x, 0) = T_0$ for $x \geq 0$ and $T(x, 0) = 0$, otherwise. Here $T_0$ is a constant. Hint: Use the Green function approach introduced in class.

(c) Express the answer of part (b) in terms of the error function.

(d) Show that for $t > 0$ and as $x \to \infty$, then $T \to T_0$ for the solution you found in part (b).

(e) Show that for $t > 0$ and as $x \to -\infty$, then $T \to 0$, for the solution you found in part (b).