Honors Calculus 252-2
Winter 2002
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Problem Set 4

to be handed in Friday February 8, 2002, in class

1. A bug is crawling radially outward at a speed C on a record player rotating at an angular velocity Ω.

   (a) Show that its velocity at a point (x, y) is given by

   $$\mathbf{v}(x,y) = \left(-\Omega y + \frac{Cx}{\sqrt{x^2+y^2}}, \Omega x + \frac{Cy}{\sqrt{x^2+y^2}}\right)$$

   (b) If the bug is initially at a distance D from the center and at an angle α show that its polar coordinates after a time t are

   $$r(t) = D + Ct \qquad \theta(t) = \alpha + \Omega t.$$

   Compute its cartesian coordinates $x(t), y(t)$.

   (c) Show that the bug's trajectory in (b) is a streamline of the vector field $\mathbf{v}$.

2. The electrostatic potential, Φ, is defined as $\mathbf{E} = -\nabla\Phi$. Consider an electric field,

   $$\mathbf{E} = Re^{-R^2}\hat{R},$$

   where $\hat{R}$ is the unit vector in the radial direction in a spherical coordinate system. Construct $\Phi(x, y, z)$ by doing a line integral along a radial path starting at the origin; assume $\Phi(0, 0, 0) = 0$. Verify your result by evaluating $\nabla\Phi$.

3. 8.1.5, 8.1.8(i)

4. 8.2.5, 8.2.14(i), 8.2.14(ii)