Honors Calculus 252-2

Winter 2002

Hermann Riecke

Problem Set 6

to be handed in Monday February 25, 2002, in class

1. One of Maxwell's Equations states that a charge density $\rho(x, y, z)$ generates an electric field, $\mathbf{E}$, that satisfies
$$\nabla \cdot \mathbf{E} = 4\pi\rho(x, y, z).$$

   (a) Assume that the charge density is cylindrically symmetric of the form
$$\rho(x, y, z) = Q(r) \qquad r = \sqrt{x^2 + y^2}.$$
   then by symmetry, we can expect an electric field of the form
$$\mathbf{E} = F(r)\hat{r} \qquad \hat{r} = \frac{x\hat{i} + y\hat{j}}{r}.$$
   Show that
$$\frac{\partial F}{\partial r} + \frac{1}{r}F = 4\pi Q.$$

   (b) Verify the identity
$$\frac{\partial F}{\partial r} + \frac{1}{r}F = \frac{1}{r}\frac{\partial (rF)}{\partial r}.$$
   Use this identity to compute the electric field for a charge distribution (a model of a wire)
$$Q(r) = e^{-r^2}.$$

   (c) Suppose that the charge distribution is spherically symmetric. Compute the analogous relationship between charge and field to that computed in part (a).

2. Verify the divergence theorem for the following vector functions $\mathbf{F}(x, y, z)$ where V is the rectangular cube bounded by the coordinate planes and the planes $x = a, y = b$ and $z = c$.
$$(a) \quad \mathbf{F} = x\ \hat{\mathbf{i}} + y\ \hat{\mathbf{j}} + z\ \hat{\mathbf{k}} \qquad (b) \quad \mathbf{F} = xy\ \hat{\mathbf{i}} + yz\ \hat{\mathbf{j}} + zx\ \hat{\mathbf{k}}$$

3. 8.4.16

4. 8.5.7