Honors Calculus 252-2
Winter 2002
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Review Sheet for Midterm

1. (10 points) Sketch the region of integration and reverse the order of integration of

$$\int_0^1 dx \int_{-\sqrt{1-x}}^{\sqrt{1-x}} f(x,y)\; dy.$$

2. (10 points) Write an integral for the area of the surface defined by $z = xy$ in the region $x^2 + y^2 \leq 1$ in (a) cartesian coordinates, and in (b) polar coordinates. DO NOT EVALUATE THE INTEGRALS.

3. (25 points) Compute the gravitational force, $\vec{F}$, exerted on a mass M at the origin by the portion of a spherical shell in the first quadrant:

$$x^2 + y^2 + z^2 = a^2 \qquad x \geq 0 \qquad y \geq 0 \qquad z \geq 0.$$

   You may assume the shell has a constant density, σ, and that the gravitational constant is G.

4. (25 points) Find the moment of inertia of a solid spherical shell of constant density ρ with interior radius a and exterior radius b about an axis through the center of the sphere.

5. (30 points) Assume that $b > a > 0$ and $d > c > 0$. Consider the region R in the first quadrant contained by the curves $y^3 = ax$, $y^3 = bx$, $x^2 = cy$ and $x^2 = dy$. Make a change of coordinates $u = u(x,y)$ and $v = v(x,y)$ that turns this region into a rectangular one in the (u,v)-plane. Use this coordinate transformation to determine the area of the region R.

6. (7.2,19) What is the volume contained by the three surfaces

$$z = 0, \qquad z = \frac{x^2}{p^2} + \frac{y^2}{q^2}, \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1? \tag{1}$$

7. (7.2,30) With $m! = 2\int_0^\infty x^{2m+1} e^{-x^2}\, dx$ and $n! = 2\int_0^\infty y^{2n+1} e^{-y^2}\, dy$ form the double integral $m!n!$. By transforming to polar coordinates show that

$$\int_0^{\pi/2} \cos^{2m+1}\theta \, \sin^{2n+1}\theta \, d\theta = \frac{m!n!}{2(m+n+1)!}. \tag{2}$$

8. (7.3,22) A spherical planet has an atmosphere in which the density decreases exponentially with height above the surface. Find the total mass of the atmosphere.

9. (7.4,9) Find the area of that portion of the sphere $x^2 + y^2 + z^2 = c^2$ that is within the paraboloid $z = (x^2 + y^2)/(2c)$.

10. Consider a solid body with constant density in the shape of a silo, i.e. consisting of a cylinder with height h with a hemisphere on top of it. Find the center of mass of this body and relate it to the center of mass of the cylinder and of the hemisphere that make up the body.

Note: The problems that have points indicated constituted a midterm from a previous year. The other problems are additional practice problems.