ESAM 311-1 Methods in Applied Mathematics

Fall Quarter 2007

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Problem Set 1 **Due Friday October 12, 2007**

1. From the lecture notes do 1.11.7

2. Nerve cells (neurons) receive input from other neurons and can be excited by that input if it surpasses a certain threshold. The central quantity for the characterization of a neuron is the electrical voltage V across the membrane of the neuron. If the neuron is excited, it generates a voltage spike ('action potential') that is transmitted to other neurons, which can then become excited in turn.
   In a very simple model ('integrate-and-fire neuron') neurons are described by a first-order equation for the voltage V

   $$\tau \frac{dV}{dt} = -V + I(t),$$

   where $I(t)$ represents the input the neuron receives from other neurons. If $V > V_{threshold}$ at any time t_{spike} the model neuron generates an action potential and its voltage is reset to $V(t^+_{spike}) = 0$.
   Assume that initially the neuron is at rest, $V(t = 0) = 0$.

   (a) For $0 \le t \le T$ the neuron receives an input $I(t) = I_0$, which is turned off again at $t = T$. Calculate $V(t)$ for $0 \le t \le T$ and for $t \ge T$. Is there an input strength I_0 for which the neuron will generate a spike due to that input?

   (b) Somewhat more realistically the input from other neurons is given by

   $$I(t) = I_0 t e^{-\alpha t} \qquad \text{for} \qquad t \ge 0.$$

   Calculate $V(t)$ for $t \ge 0$.

3. From the lecture notes do the following problems:
   1.11.11, 1.11.12, 1.11.13.

4. Do problems:
   1.11.19, 1.11.20, 1.11.22, 1.11.27, 1.11.29.