

ESAM 311-1 Methods in Applied Mathematics

Fall Quarter 2007

Hermann Riecke

Midterm Review Sheet

For Monday November 5, 2007

1. Solve the initial value problem

$$y'' + y = -3 \cos 2x - 2 \sin x + 1, \quad y(0) = 2, \quad y'(0) = 2.$$

2. Solve the boundary value problem

$$x^2 y'' - 2xy' = 1 - 2y, \quad 1 < x < 2, \quad y(1) = \frac{1}{2}, \quad y'(2) = 3.$$

3. Verify that $y_1(x) = x^2$ is a solution of the equation

$$x^2 y'' + (x^2 - 4x) y' + (6 - 2x) y = 0$$

and solve the initial value problem

$$x^2 y'' + (x^2 - 4x) y' + (6 - 2x) y = 2x^4 e^x, \quad y(1) = e, \quad y'(1) = 3e.$$

4. Solve the boundary value problem

$$y'' + y = x^5 \sin x, \quad 0 < x < \pi, \quad y(0) = 0, \quad y(\pi) = 0.$$

Hint: Before trying to find a particular solution of the equation, think carefully about the boundary value problem.

5. Use Frobenius theory to compute one of the two linearly independent solutions of the equation

$$y'' + 2\frac{1}{x}y' + y = 0.$$

Can you write this solution in terms of elementary functions? Does the other linearly independent solution also have the form of a power series? Why? Why not?