# Methods of Nonlinear Analysis D-12

# Fall 1999


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# Homework Assignment 5

November 4, 1999

Due Monday November 8, 1999.

1. Consider the group $O(2)$ of $2x2$ matrices $R_\theta$ and $\kappa$ acting on the four-dimensional space $V$ of $2x2$ matrices $A$ by similarity,$\gamma \cdot A = \gamma^{-1} A \gamma$ for any $\gamma \in O(2)$ .

   a) Show that $V = V_1 \oplus V_2 \oplus V_3$ where

$$V_1 = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \qquad V_2 = \begin{pmatrix} 0 & -b \\ b & 0 \end{pmatrix} \qquad V_3 = \begin{pmatrix} c & d \\ d & -c \end{pmatrix}.$$

   b) Show that on $V_1$ the action is trivial.

   c) Show that on $V_2$ the action is given by $-I$ for the elements of $O(2)$ that contain reflections and $I$ for those that do not.

   d) Show that on $V_3$ the action is isomorphic to the standard action on $\mathrm{R}^2 \equiv \mathrm{C}$

2. Consider the group $O(2)$ of $2x2$ matrices $R_\theta$ and $\kappa$. Show that any element of $O(2)$ can be represented as $\kappa R_\theta$ with some $\theta$ .