Methods of Nonlinear Analysis D-12

Fall 1999

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Homework Assignment 6

Due Monday November 15, 1999.

1. Let $SO(2)$ act on C^n by

$$\zeta\,(z_1, z_2, ..., z_n) = (z_1 e^{i\theta}, z_2 e^{i\theta}, ..., z_n e^{i\theta}).$$

   Show that a Hilbert basis is given by

$$\{\mathrm{Re}(z_j z_k^*), \mathrm{Im}(z_j z_k^*\}.$$

2. Let $SO(2)$ act on $\mathbf{C}^2$ by

$$\zeta(z_1, z_2) = (e^{ik\theta} z_1, e^{il\theta} z_2),$$

   with k and l coprime. Show that a Hilbert basis is given by

$$\{\mathrm{Re}(z_1^l (z_2^*)^k), \mathrm{Im}(z_1^l (z_2^*)^k, |z_1|^2, |z_2|^2\}.$$

3. Show that the most general $\mathbf{D}_n-$equivariant function $f : \mathbf{C} \to \mathbf{C}$ is given by

$$f(z) = p(u,v)\, z + q(u,v)\, (z^*)^{n-1},$$

   where the invariants u and v are given by

$$u = |z|^2 \qquad \text{and} \qquad v = z^n + (z^*)^n.$$