Methods of Nonlinear Analysis D-12

Fall 1999

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Homework Assignment 8

Due Monday December 6, 1999.

1. Consider the unfolding of the pitch-fork bifurcation (in the absence of any symmetry). Using a shift $x = x_0 + \tilde{x}$ we showed in class that the bifurcation can be unfolded using two parametes,

$$\dot{\tilde{x}} = \tilde{\epsilon} + \tilde{\mu}\,\tilde{x} - \tilde{x}^3,$$

where we assume that the cubic term is saturating.

a) Sketch all topologically different bifurcation diagrams that are obtained by following linear paths in the (ϵ, μ)-parameter space. Show, in particular, that one also obtains a transcritical bifurcation for certain paths.

b) Considering experimental limitations one might, in particular, be interested in paths that involve only a *single* distinguished parameter, μ say. Show that in that case a universal unfolding requires also the quadratic term,

$$\dot{x} = \epsilon + \mu x + \nu x^2 - x^3,$$

i.e. show that all the bifurcation diagrams that are obtained in a) can be obtained by scanning μ alone if ϵ and ν have suitable (small) values, but not if $\nu = 0$.