Methods of Nonlinear Analysis D-12
Winter 2000
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Homework Assignment3

Due Thursday, January 27, 2000.

1. Oblique Rolls in Axially Anisotropic System

Consider a two-dimensional system with axial anisotropy, i.e. a system which is reflection symmetric with respect to the x- and with respect to the $y-$ axis, but which does not possess rotation symmetry. In class we derived the Ginzburg-Landau equation for stripe patterns (rolls) in such a system if they are aligned with one of the axes of the system. Assume now that the initial instability is to rolls that are rotated by an angle $0 < \theta_c < \pi/2$ relative to the axes.

a) Use symmetry and genericity arguments to derive a description of this system within a Ginzburg-Landau-type framework (allowing slowly varying space coordinates). Note that for this case θ_c is assumed to bounded away from 0.

b) Depending on the values of the cubic coefficients, determine all spatially periodic stationary solutions of the resulting equation(s) and describe their optical appearance. Ignoring side-band instabilities which solutions are stable? Assume that the bifurcations are supercritical.

c) Now consider the intermediate case in which the critical angle θ_c is close to 0. In this case, depending on a parameter the linear growth rate $\sigma(q,p)$ characterizing the stability of the basic state changes from having a maximum at $(q = q_c^{(1)}, 0)$ to having a saddle point there with new maxima appearing at $(q_c^{(2)}, \pm p_c)$. Give a Ginzburg-Landau-type equation in which the slight obliqueness of the rolls ($p_c^2 \neq 0$) is captured by a slow $Y-$ coordinate.

d) In the equation derived in c) assume now that $p_c^2 \neq 0$. By considering the limit of small amplitudes at *fixed* p_c derive the equations derived in a) from that obtained in c). Ignoring side-band instabilities, which solutions are stable?