Methods of Nonlinear Analysis D-12
Winter 2000
Hermann Riecke
Homework Assignment 4

Due Thursday, February 10, 2000.

1. Mean-Field Ginzburg-Landau Equations

Consider the following nonlinear wave equation

$$\partial_t^2\psi - \partial_x^2\psi = (\mu-1)\partial_t\psi - 2\partial_t\partial_x^2\psi - \partial_t\partial_x^4\psi + \alpha\psi^3 + \beta\psi^2\partial_t\psi$$

on a periodic domain of length $L \gg 1$.
a) Perform a linear stability analysis of the trivial state $\psi = 0$. Determine the phase and the group velocity of the linear waves as a function of the wavenumber.
b) For small μ derive the mean-field Ginzburg-Landau equations.
c) Within the mean-field Ginzburg-Landau equations, determine the stability of standing waves with wavenumbers $q - q_c = O(\mu^{1/2})$. Distinguish the cases of zero and of non-zero Floquet parameter K. Consider only the long-wave limit $K \ll 1$.