Methods of Nonlinear Analysis D-12
Winter 2000
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Homework Assignment 5

Due Thursday, February 17, 2000.

1. Perturbed Korteweg-deVries Equation

The dissipatively perturbed KdV-equation is given by

$$\partial_t A = -\sigma_3 \partial_x^3 A + c_2 \partial_x A^2 + \nu \left(-\mu \partial_x^2 A - \partial_x^4 A + \hat{c}_2 \partial_x^2 A^2\right), \qquad \nu \ll 1. \tag{1}$$

As with the SH-equation, exact periodic solutions are not available. Thus, although the derivation of the KdV-equation from microscopic equations like Navier-Stokes etc. amounts to a substantial simplification of the problem, it is still useful to perform a weakly nonlinear analysis of this reduced equation. Note, that the perturbed KdV-equation is obtained in a long-wave expansion. Therefore $\nu \ll 1$.

a) Derive a Ginzburg-Landau equation for spatially periodic solutions with wavenumber q. To treat the smallness of ν properly, perform the derivation in a double expansion in ν and ϵ, where ϵ measures the small amplitude. Thus, at each order in ϵ go to $O(\nu)$. This gives you then the coefficients in the Ginzburg-Landau equation in an expansion in ν.

b) As we will see later, the perturbed KdV-equation (1) describes the phase dynamics of traveling waves near their Eckhaus instability. Is this bifurcation super- or subcritical?

c) In the absence of the dispersive terms involving σ_3 and c_2 (1) describes the phase dynamics of steady patterns near the Eckhaus instability. Is that bifurcation super- or subcritical?

2. Patterns in Films

Consider a thin film of liquid on a plate that is heated from below. The upper surface of the film is exposed to air and can be deformed. Assume that this system undergoes an instability. In each of the three cases listed below use symmetry and scaling arguments to identify the form of the evolution equations that you expect to arise in a weakly nonlinear description of the instability. Consider only the one-dimensional case.

a) The instability involves a real eigenvalue at a finite wavenumber. This is the case for Marangoni convection , i.e. surface-tension-driven convection (e.g. [1]).

b) The instability involves a complex eigenvalue at vanishing wavenumer.

c) The instability involves a complex eigenvalue at a finite wavenumber

Note: your result will illustrate that

i) conservation laws imply slow long-wave dynamics and

ii) conserved quantities have to be included in a weakly nonlinear description even if they do not go unstable by themselves.

References

[1] A.A. Golovin, A.A. Nepomnyashchy, L.M. Pismen, and H. Riecke. Steady and oscillatory side-band instabilities in Marangoni convection with deformable interface. *Physica D*, 106:131–147, 1997.