Methods of Nonlinear Analysis D-12
Winter 2000
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Homework Assignment 6

Due Thursday, February 24, 2000.

1. Long-Wave Marangoni Convection

Consider a film of liquid on a poorly conducting plate and exposed to air. The liquid is heated from below. If the film is sufficiently thin buoyancy can be neglected in the liquid and convection is only driven by gradients in the surface tension. Assume the Prandtl number is very high. As in the case of Rayleigh-Benard convection, by taking the curl twice of the momentum equation one obtains then for the fluid equations in the bulk the dimensionless equations

$$\Delta^2 W = 0 \quad (1)$$
$$\partial_t\theta + U\partial_x\theta - W = \Delta\theta \quad (2)$$
$$\partial_x U + \partial_z W = 0. \quad (3)$$

At the bottom ($z = 0$) the boundary conditions are given by

$$W = 0 \quad (4)$$
$$U = 0 \quad (5)$$
$$\partial_z\theta = Bi_b\,\theta, \quad (6)$$

whereas at the top ($z = 1$) the boundary conditions are given by

$$W = 0 \quad (7)$$
$$\partial_z^2 W = Ma\,\partial_x^2\theta \quad (8)$$
$$\partial_z\theta = -Bi_t\,\theta. \quad (9)$$

Eqs.(4,5) express impermeability and no-slip, respectively. Assuming that the free surface remains flat leads to (7). The stress exerted by gradients in the surface tension induces vertical gradients of the horizontal velocity implying (8). The heat boundary conditions (6,9) assume that the boundaries are poor conductors, thus the Biot numbers $Bi_{b,t}$ are small.

Assignment:
Derive a long-wave evolution equation describing convection in this system in the limit of small Biot numbers.

To do so, choose the scaling taken in class

$$(U, W, \theta) = (\epsilon U_0 + ..., \epsilon^2 W_0 + ..., \theta_0 + ...)$$

and

$$Bi_{t,b} = \epsilon^4\,\beta_{t,b} \qquad Ma = Ma_c\,(1 + \epsilon^2 M_2),$$

$$X = \epsilon x \qquad T = \epsilon^4 t.$$

Discuss the effect of non-vanishing Biot numbers on the neutral curve for the appearance of steady patterns.