Nr 3,3,2

higher orders: $y = y_0 + \epsilon y_1 + \cdots$

outer: $O(\epsilon)$:

$$x^2 y_1' - y_1 = -y_0'' = -\left[\frac{2}{x^3} 2e\, e^{-\frac{1}{x}} - \frac{2e}{x^2}\left(-\frac{1}{x^2}\right) e^{-\frac{1}{x}}\right]$$

$$\left(y_0' = -\frac{1}{x^2} y_0\right)$$

$$= -\frac{4e^{1-\frac{1}{x}}}{x^3} - \frac{2}{x^4} e^{1-\frac{1}{x}}$$

$$y_1' - \frac{1}{x^2} y_1 = -\frac{4}{x^5} e^{1-\frac{1}{x}} - \frac{2}{x^6} e^{1-\frac{1}{x}}$$

$$e^{-\frac{1}{x}} \frac{d}{dx}\left(e^{+\frac{1}{x}} y_1\right) = e^{-\frac{1}{x}}\left[-\frac{4e}{x^5} - \frac{2e}{x^6}\right]$$

$$y_1 = e^{-\frac{1}{x}}\left[\frac{e}{x^4} + \frac{2}{5}\frac{e}{x^5} + C_4\right]$$

inner: $Y_{1/2}'' - Y_{1/2} = -\xi^2 Y_0' = +\xi^2 e^{-\xi}$

$$Y_{1/2} = C_5 e^{-\xi} + C_6 \xi^3 e^{-\xi} + C_7 \xi^2 e^{-\xi} + C_8 \xi e^{-\xi}$$

$$e^{-\xi}\Big\{+C_5 + C_6\left[6\xi + 2\cdot 3\xi^2(-1) + \xi^3\right] +$$

$$+ C_7\left[2 + 2\cdot 2\xi(-1) + \xi^2\right] + C_8\left[2(-1) + \xi\right]\Big\} -$$

$$- e^{-\xi}\left[C_5 + \xi^3 C_6 + \xi^2 C_7 + \xi C_8 + \xi^2\right] = 0$$

$$C_5 + 2C_7 - 2C_8 - C_5 = 0 \Rightarrow C_7 = C_8$$

$$6C_6 - 4C_7 + C_8 - C_8 = 0 \Rightarrow C_6 = \frac{2}{3} C_7$$

$$-6C_6 + C_7 - C_7 + 1 = 0 \Rightarrow C_6 = \frac{1}{6}$$

$$C_6 - C_6 = 0$$

check algebra → maple

$$Y_{1/2} = e^{-\xi}\left\{C_5 + \frac{1}{6}\xi^3 + \frac{1}{4}\xi^2 + \frac{1}{4}\xi\right\}$$