Asymptotic and Perturbation Methods in Applied Mathematics
ESAM 420-1

Fall 2002

Hermann Riecke

Problem Set 3

Due November 12, 2002.

1. The following problem was suggested by Friedrichs (1942) as a simple analogue to the structure of a boundary layer in a viscous fluid. Consider

$$\epsilon y'' + y' = \alpha$$

$$y(0) = 0, \quad y(1) = 1.$$

where $\epsilon \ll 1$.

(a) Determine the first terms in the inner and outer expansions, and write down a composite expansion of the solution of this problem. What happens when $\alpha = 1$?

(b) Derive a two-term composite expansion of the solution of the problem.

2. Determine the first terms in the inner and outer expansions for the following problem:

$$\epsilon y'' - (2x+1)y' + 2y = 0$$

$$y(0) = 1, \quad y(1) = 0.$$

Construct a first order uniformly valid expansion for $y(x)$ and sketch the solution.

3. Construct a uniformly valid expansion, correct to $O(\epsilon)$, for the problem

$$\epsilon y'' + x^2 y' - y = 0$$

$$y(0) = 1, \quad y(1) = 2.$$

4. A one dimensional heat transfer problem is governed by

$$\epsilon \frac{d^2T}{dx^2} + x\frac{dT}{dx} - xT = 0$$

$$T(0) = T_0, \quad T(1) = T_1,$$

where T is the temperature. Determine a first order uniformly valid expansion for $\epsilon \ll 1$.