```
> restart;
> gl:=omega^2*diff(u(phi),phi,phi)+u(phi)-omega^2*epsilon*u(phi)*(di
  ff(u(phi),phi))^2;
```

$$gl := \omega^2 \left(\frac{\partial^2}{\partial \phi^2} u(\phi) \right) + u(\phi) - \omega^2 \, \varepsilon \, u(\phi) \left(\frac{\partial}{\partial \phi} u(\phi) \right)^2$$

```
> defu:=u(phi)=u0(phi)+epsilon*u1(phi)+epsilon^2*u2(phi);
```

$$defu := u(\phi) = u0(\phi) + \varepsilon \, u1(\phi) + \varepsilon^2 \, u2(\phi)$$

```
> defom:=omega=omega[0]+epsilon*omega[1]+epsilon^2*omega[2];
```

$$defom := \omega = \omega_0 + \varepsilon \, \omega_1 + \varepsilon^2 \, \omega_2$$

```
> subs([defu,defom],gl);
```

$$(\omega_0 + \varepsilon \, \omega_1 + \varepsilon^2 \, \omega_2)^2 \left(\frac{\partial^2}{\partial \phi^2} (u0(\phi) + \varepsilon \, u1(\phi) + \varepsilon^2 \, u2(\phi)) \right) + u0(\phi) + \varepsilon \, u1(\phi) + \varepsilon^2 \, u2(\phi)$$

$$- (\omega_0 + \varepsilon \, \omega_1 + \varepsilon^2 \, \omega_2)^2 \, \varepsilon \, (u0(\phi) + \varepsilon \, u1(\phi) + \varepsilon^2 \, u2(\phi)) \left(\frac{\partial}{\partial \phi} (u0(\phi) + \varepsilon \, u1(\phi) + \varepsilon^2 \, u2(\phi)) \right)^2$$

```
> gle:=convert(taylor(%,epsilon=0,3),polynom);
```

$$gle := \omega_0^2 \left(\frac{\partial^2}{\partial \phi^2} u0(\phi) \right) + u0(\phi)$$

$$+ \left(\omega_0^2 \left(\frac{\partial^2}{\partial \phi^2} u1(\phi) \right) + 2 \, \omega_0 \, \omega_1 \left(\frac{\partial^2}{\partial \phi^2} u0(\phi) \right) - \omega_0^2 \, u0(\phi) \left(\frac{\partial}{\partial \phi} u0(\phi) \right)^2 + u1(\phi) \right) \varepsilon + \left(\omega_0^2 \left(\frac{\partial^2}{\partial \phi^2} u2(\phi) \right) \right.$$

$$\left. + 2 \, \omega_0 \, \omega_1 \left(\frac{\partial^2}{\partial \phi^2} u1(\phi) \right) + (2 \, \omega_0 \, \omega_2 + \omega_1^2) \left(\frac{\partial^2}{\partial \phi^2} u0(\phi) \right) + u2(\phi) - 2 \, \omega_0^2 \, u0(\phi) \left(\frac{\partial}{\partial \phi} u0(\phi) \right) \left(\frac{\partial}{\partial \phi} u1(\phi) \right) \right.$$

$$\left. - (\omega_0^2 \, u1(\phi) + 2 \, \omega_0 \, \omega_1 \, u0(\phi)) \left(\frac{\partial}{\partial \phi} u0(\phi) \right)^2 \right) \varepsilon^2$$

```
> gl0:=coeff(gle,epsilon,0);
> defu0:=u0(phi)=A[0]*(exp(I*phi)+exp(-I*phi));
> defom0:=omega[0]=1;
> simplify(subs([defu0,defom0],gl0));
```

$$gl0 := \omega_0^2 \left(\frac{\partial^2}{\partial \phi^2} u0(\phi) \right) + u0(\phi)$$

$$defu0 := u0(\phi) = A_0 \, (\mathbf{e}^{(I\phi)} + \mathbf{e}^{(-I\phi)})$$

$$defom0 := \omega_0 = 1$$

$$0$$

```
> 
> gl1:=coeff(gle,epsilon,1);
```

$$gl1 := \omega_0^2 \left(\frac{\partial^2}{\partial \phi^2} u1(\phi) \right) + 2 \, \omega_0 \, \omega_1 \left(\frac{\partial^2}{\partial \phi^2} u0(\phi) \right) - \omega_0^2 \, u0(\phi) \left(\frac{\partial}{\partial \phi} u0(\phi) \right)^2 + u1(\phi)$$

```
> gl1e:=expand(subs([defu0,defom0],%));
```

$$gl1e := \left(\frac{\partial^2}{\partial \phi^2}\,\mathrm{u1}(\phi)\right) - 2\,\omega_1\,A_0\,\mathbf{e}^{(I\phi)} - \frac{2\,\omega_1\,A_0}{\mathbf{e}^{(I\phi)}} + A_0{}^3\,(\mathbf{e}^{(I\phi)})^3 - \mathbf{e}^{(I\phi)}\,A_0{}^3 - \frac{A_0{}^3}{\mathbf{e}^{(I\phi)}} + \frac{A_0{}^3}{(\mathbf{e}^{(I\phi)})^3} + \mathrm{u1}(\phi)$$

```
> defu1:=u1(phi)=A[1]*exp(I*phi)+As[1]*exp(-I*phi)+B[1]*exp(3*I*phi)
  +Bs[1]*exp(-3*I*phi):
> defu1:=subs([A[1]=0,As[1]=0],defu1);
```

$$defu1 := \mathrm{u1}(\phi) = B_1\,\mathbf{e}^{(3\,I\phi)} + Bs_1\,\mathbf{e}^{(-3\,I\phi)}$$

```
> subs(defu1,gl1e):
> gl1ee:=expand(%);
```

$$gl1ee := -8\,B_1\,(\mathbf{e}^{(I\phi)})^3 - \frac{8\,Bs_1}{(\mathbf{e}^{(I\phi)})^3} - 2\,\omega_1\,A_0\,\mathbf{e}^{(I\phi)} - \frac{2\,\omega_1\,A_0}{\mathbf{e}^{(I\phi)}} + A_0{}^3\,(\mathbf{e}^{(I\phi)})^3 - \mathbf{e}^{(I\phi)}\,A_0{}^3 - \frac{A_0{}^3}{\mathbf{e}^{(I\phi)}} + \frac{A_0{}^3}{(\mathbf{e}^{(I\phi)})^3}$$

```
> coeff(gl1ee,exp(I*phi),-3):
> defBs1:=Bs[1]=solve(%,Bs[1]);
```

$$defBs1 := Bs_1 = \frac{1}{8}\,A_0{}^3$$

```
> coeff(gl1ee,exp(I*phi),3):
> defB1:=B[1]=solve(%,B[1]);
```

$$defB1 := B_1 = \frac{1}{8}\,A_0{}^3$$

```
> coeff(gl1ee,exp(I*phi),1):
> defom1:=omega[1]=solve(%,omega[1]);
```

$$defom1 := \omega_1 = -\frac{1}{2}\,A_0{}^2$$

```
> subs([defom1,defB1,defBs1],gl1ee);
```

$$0$$

```
> gl2:=coeff(gle,epsilon,2);
```

$$gl2 := \omega_0{}^2\left(\frac{\partial^2}{\partial \phi^2}\,\mathrm{u2}(\phi)\right) + 2\,\omega_0\,\omega_1\left(\frac{\partial^2}{\partial \phi^2}\,\mathrm{u1}(\phi)\right) + (2\,\omega_0\,\omega_2 + \omega_1{}^2)\left(\frac{\partial^2}{\partial \phi^2}\,\mathrm{u0}(\phi)\right) + \mathrm{u2}(\phi)$$

$$- 2\,\omega_0{}^2\,\mathrm{u0}(\phi)\left(\frac{\partial}{\partial \phi}\,\mathrm{u0}(\phi)\right)\left(\frac{\partial}{\partial \phi}\,\mathrm{u1}(\phi)\right) - (\omega_0{}^2\,\mathrm{u1}(\phi) + 2\,\omega_0\,\omega_1\,\mathrm{u0}(\phi))\left(\frac{\partial}{\partial \phi}\,\mathrm{u0}(\phi)\right)^2$$

```
> subs([defu0,defu1,defom0,defom1],%):
> subs([defB1,defBs1],%):
> gl2e:=expand(%);
```

$$gl2e := \left(\frac{\partial^2}{\partial \phi^2}\,\mathrm{u2}(\phi)\right) - \frac{1}{8}\,A_0{}^5\,(\mathbf{e}^{(I\phi)})^3 - \frac{1}{8}\,\frac{A_0{}^5}{(\mathbf{e}^{(I\phi)})^3} - 2\,A_0\,\omega_2\,\mathbf{e}^{(I\phi)} - \frac{2\,A_0\,\omega_2}{\mathbf{e}^{(I\phi)}} + \frac{1}{8}\,A_0{}^5\,\mathbf{e}^{(I\phi)} + \frac{\frac{1}{8}\,A_0{}^5}{\mathbf{e}^{(I\phi)}}$$

$$+\mathrm{u2}(\phi)+\frac{7}{8}A_0^{\,5}\left(\mathbf{e}^{(I\phi)}\right)^5+\frac{\frac{7}{8}A_0^{\,5}}{\left(\mathbf{e}^{(I\phi)}\right)^5}$$

```
> defu2:=u2(phi)=C[2]*exp(5*I*phi)+Cs[2]*exp(-5*I*phi)+B[2]*exp(3*I*
  phi)+Bs[2]*exp(-3*I*phi);
```

$$defu2 := \mathrm{u2}(\phi) = C_2\,\mathbf{e}^{(5I\phi)} + Cs_2\,\mathbf{e}^{(-5I\phi)} + B_2\,\mathbf{e}^{(3I\phi)} + Bs_2\,\mathbf{e}^{(-3I\phi)}$$

```
> subs(defu2,gl2e):
> gl2ee:=expand(%);
```

$$gl2ee := -24\,C_2\left(\mathbf{e}^{(I\phi)}\right)^5 - \frac{24\,Cs_2}{\left(\mathbf{e}^{(I\phi)}\right)^5} - 8\,B_2\left(\mathbf{e}^{(I\phi)}\right)^3 - \frac{8\,Bs_2}{\left(\mathbf{e}^{(I\phi)}\right)^3} - \frac{1}{8}A_0^{\,5}\left(\mathbf{e}^{(I\phi)}\right)^3 - \frac{1}{8}\frac{A_0^{\,5}}{\left(\mathbf{e}^{(I\phi)}\right)^3}$$

$$-2\,A_0\,\omega_2\,\mathbf{e}^{(I\phi)} - \frac{2\,A_0\,\omega_2}{\mathbf{e}^{(I\phi)}} + \frac{1}{8}A_0^{\,5}\,\mathbf{e}^{(I\phi)} + \frac{\frac{1}{8}A_0^{\,5}}{\mathbf{e}^{(I\phi)}} + \frac{7}{8}A_0^{\,5}\left(\mathbf{e}^{(I\phi)}\right)^5 + \frac{\frac{7}{8}A_0^{\,5}}{\left(\mathbf{e}^{(I\phi)}\right)^5}$$

```
> coeff(gl2ee,exp(I*phi),1):
> defom2:=omega[2]=solve(%,omega[2]);
```

$$defom2 := \omega_2 = \frac{1}{16}A_0^{\,4}$$

```
> coeff(gl2ee,exp(I*phi),3):
> defB2:=B[2]=solve(%,B[2]);
```

$$defB2 := B_2 = -\frac{1}{64}A_0^{\,5}$$

```
> coeff(gl2ee,exp(I*phi),-3):
> defBs2:=Bs[2]=solve(%,Bs[2]);
```

$$defBs2 := Bs_2 = -\frac{1}{64}A_0^{\,5}$$

```
> coeff(gl2ee,exp(I*phi),5):
> defC2:=C[2]=solve(%,C[2]);
```

$$defC2 := C_2 = \frac{7}{192}A_0^{\,5}$$

```
> coeff(gl2ee,exp(I*phi),-5):
> defCs2:=Cs[2]=solve(%,Cs[2]);
```

$$defCs2 := Cs_2 = \frac{7}{192}A_0^{\,5}$$

```
> subs([defu0,defu1,defu2],defu):
> subs([defB1,defBs1,defB2,defBs2,defC2,defCs2],%):
> simplify(%);
```

$$\mathrm{u}(\phi) = \frac{7}{96}\varepsilon^2 A_0^{\,5}\cos(5\,\phi) + 2\,A_0\cos(\phi) + \frac{1}{4}\cos(3\,\phi)\,\varepsilon\,A_0^{\,3} - \frac{1}{32}\cos(3\,\phi)\,\varepsilon^2 A_0^{\,5}$$

```
>
```