```
> restart;
> gl:=omega^2*diff(u(phi),phi,phi)*(1+4*u(phi)^2)+u(phi)+4*omega^2*u
  (phi)*(diff(u(phi),phi))^2;
```

$$gl := \omega^2 \left(\frac{\partial^2}{\partial \phi^2} u(\phi)\right)(1 + 4\, u(\phi)^2) + u(\phi) + 4\, \omega^2\, u(\phi) \left(\frac{\partial}{\partial \phi} u(\phi)\right)^2$$

```
> defu:=u(phi)=epsilon*u1(phi)+epsilon^3*u3(phi);
```

$$defu := u(\phi) = \varepsilon\, u1(\phi) + \varepsilon^3\, u3(\phi)$$

```
> defom:=omega=omega[0]+epsilon^2*omega[2];
```

$$defom := \omega = \omega_0 + \varepsilon^2\, \omega_2$$

```
> subs([defu,defom],gl);
```

$$(\omega_0 + \varepsilon^2\, \omega_2)^2 \left(\frac{\partial^2}{\partial \phi^2} (\varepsilon\, u1(\phi) + \varepsilon^3\, u3(\phi))\right)(1 + 4\, (\varepsilon\, u1(\phi) + \varepsilon^3\, u3(\phi))^2) + \varepsilon\, u1(\phi) + \varepsilon^3\, u3(\phi)$$
$$+ 4\, (\omega_0 + \varepsilon^2\, \omega_2)^2\, (\varepsilon\, u1(\phi) + \varepsilon^3\, u3(\phi)) \left(\frac{\partial}{\partial \phi} (\varepsilon\, u1(\phi) + \varepsilon^3\, u3(\phi))\right)^2$$

```
> gle:=convert(taylor(%,epsilon=0,4),polynom);
```

$$gle := \left(\omega_0^{\,2} \left(\frac{\partial^2}{\partial \phi^2} u1(\phi)\right) + u1(\phi)\right)\varepsilon + \left(4\, \omega_0^{\,2} \left(\frac{\partial^2}{\partial \phi^2} u1(\phi)\right) u1(\phi)^2 + \omega_0^{\,2} \left(\frac{\partial^2}{\partial \phi^2} u3(\phi)\right)\right.$$
$$\left. + 2\, \omega_0\, \omega_2 \left(\frac{\partial^2}{\partial \phi^2} u1(\phi)\right) + u3(\phi) + 4\, \omega_0^{\,2}\, u1(\phi) \left(\frac{\partial}{\partial \phi} u1(\phi)\right)^2\right)\varepsilon^3$$

```
> gl1:=coeff(gle,epsilon,1);
> defu1:=u1(phi)=A[1]*(exp(I*phi)+exp(-I*phi));
> defom0:=omega[0]=1;
> simplify(subs([defu1,defom0],gl1));
```

$$gl1 := \omega_0^{\,2} \left(\frac{\partial^2}{\partial \phi^2} u1(\phi)\right) + u1(\phi)$$

$$defu1 := u1(\phi) = A_1\, (\mathbf{e}^{(I\, \phi)} + \mathbf{e}^{(-I\, \phi)})$$

$$defom0 := \omega_0 = 1$$

$$0$$

```
>
>  gl3:=coeff(gle,epsilon,3);
```

$$gl3 := 4\, \omega_0^{\,2} \left(\frac{\partial^2}{\partial \phi^2} u1(\phi)\right) u1(\phi)^2 + \omega_0^{\,2} \left(\frac{\partial^2}{\partial \phi^2} u3(\phi)\right) + 2\, \omega_0\, \omega_2 \left(\frac{\partial^2}{\partial \phi^2} u1(\phi)\right) + u3(\phi)$$
$$+ 4\, \omega_0^{\,2}\, u1(\phi) \left(\frac{\partial}{\partial \phi} u1(\phi)\right)^2$$

```
> gl3e:=expand(subs([defu1,defom0],%));
```

$$gl3e :=$$

$$-8 A_1^{3} (e^{(I\phi)})^{3} - 8 A_1^{3} e^{(I\phi)} - \frac{8 A_1^{3}}{e^{(I\phi)}} - \frac{8 A_1^{3}}{(e^{(I\phi)})^{3}} + \left(\frac{\partial^2}{\partial\phi^2} \mathrm{u3}(\phi)\right) - 2\,\omega_2 A_1 e^{(I\phi)} - \frac{2\,\omega_2 A_1}{e^{(I\phi)}} + \mathrm{u3}(\phi)$$

```
> defu3:=u3(phi)=A[3]*exp(I*phi)+As[3]*exp(-I*phi)+B[3]*exp(3*I*phi)
  +Bs[3]*exp(-3*I*phi):
> defu3:=subs([A[3]=0,As[3]=0],defu3);
```

$$defu3 := \mathrm{u3}(\phi) = B_3\, e^{(3 I\phi)} + Bs_3\, e^{(-3 I\phi)}$$

```
> subs([defom0,defu3],gl3e):
> gl3ee:=expand(%);
```

$gl3ee :=$

$$-8 A_1^{3} (e^{(I\phi)})^{3} - 8 A_1^{3} e^{(I\phi)} - \frac{8 A_1^{3}}{e^{(I\phi)}} - \frac{8 A_1^{3}}{(e^{(I\phi)})^{3}} - 8 B_3 (e^{(I\phi)})^{3} - \frac{8 Bs_3}{(e^{(I\phi)})^{3}} - 2\,\omega_2 A_1 e^{(I\phi)} - \frac{2\,\omega_2 A_1}{e^{(I\phi)}}$$

```
>
> coeff(gl3ee,exp(I*phi),-3):
> defBs3:=Bs[3]=solve(%,Bs[3]);
```

$$defBs3 := Bs_3 = -A_1^{3}$$

```
> coeff(gl3ee,exp(I*phi),3):
> defB3:=B[3]=solve(%,B[3]);
```

$$defB3 := B_3 = -A_1^{3}$$

```
> coeff(gl3ee,exp(I*phi),1):
> defom2:=omega[2]=solve(%,omega[2]);
```

$$defom2 := \omega_2 = -4 A_1^{2}$$

```
>
> subs([defB3,defBs3,defom2],gl3ee);
```

$$0$$

```
>
> subs([defu1,defu3],defu);
```

$$\mathrm{u}(\phi) = \varepsilon A_1 (e^{(I\phi)} + e^{(-I\phi)}) + \varepsilon^3 (B_3\, e^{(3 I\phi)} + Bs_3\, e^{(-3 I\phi)})$$

```
> subs([defu1,defB3,defBs3],%);
```

$$\mathrm{u}(\phi) = \varepsilon A_1 (e^{(I\phi)} + e^{(-I\phi)}) + \varepsilon^3 (-A_1^{3} e^{(3 I\phi)} - A_1^{3} e^{(-3 I\phi)})$$

```
> simplify(%);
```

$$\mathrm{u}(\phi) = -2\,\varepsilon^3 A_1^{3} \cos(3\,\phi) + 2\,\varepsilon A_1 \cos(\phi)$$

```
>
>
```