This program calculates the amplitude eqation for a Hopf bifurcation in a 2 component reaction diffusion equation.

```
> restart;
> mkoeff:=proc(exp,var1,max1,var2,max2)
    global koeff2:
    local mk_exp, ausdruck, ivar1, mk_exp1, ivar2:
    mk_exp:=collect(exp,var1):
    for ivar1 from 0 to max1 do
      mk_exp1[ivar1]:=coeff(mk_exp,var1,ivar1):
      mk_exp1[ivar1]:=collect(mk_exp1[ivar1],var2):
      for ivar2 from 0 to max2 do
        koeff2[ivar1,ivar2]:=coeff(mk_exp1[ivar1],var2,ivar2):
        ausdruck:=var1^ivar1*var2^ivar2:
        print(ausdruck,koeff2[ivar1,ivar2]):
      od:
    od:
  end:
> abl:=proc(func,var)
  if (var=t) then
    diff(func,t)+epsilon*diff(func,t1)+epsilon**2*diff(func,t2)+
    epsilon**3*diff(func,t3)+epsilon**4*diff(func,t4)
  elif (var=x) then
    diff(func,x)+epsilon*diff(func,x1)
  elif (var=y) then
    diff(func,y)+epsilon*diff(func,y1)
  else
    diff(func,var)
  fi
  end:
> alias(x=xf(t,t2),y=yf(t,t2),A1=Af(t2),B21=B21f(t2),B22=B22f(t2),C2
  1=C21f(t2),C22=C22f(t2),A21=A21f(t2),A22=A22f(t2),A31=A31f(t2),A32
  =A32f(t2),B31=B31f(t2),B32=B32f(t2),C31=C31f(t2),C32=C32f(t2),D31=
  D31f(t2),D32=D32f(t2),A1s=Asf(t2),B21s=B21sf(t2),B22s=B22sf(t2),C2
  1s=C2xsf(t2),C22s=C22sf(t2),A21s=A21sf(t2),A22s=A22sf(t2),A31s=A31
  sf(t2),A32s=A32sf(t2),B31s=B31sf(t2),B32s=B32sf(t2),C31s=C31sf(t2)
  ,C32s=C32sf(t2),D31s=D31sf(t2),D32s=D32sf(t2));
```

$x, y, A1, B21, B22, C21, C22, A21, A22, A31, A32, B31, B32, C31, C32, \mathrm{D31}, \mathrm{D32}, A1s, B21s,$
$B22s, C21s, C22s, A21s, A22s, A31s, A32s, B31s, B32s, C31s, C32s, D31s, D32s$

```
> with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
```

`> A:=1;` with A=1 there are no DC-contributions and the results become much simpler

$$A := 1$$

```
> gl1:=-abl(x,t)+A-(B+1)*x+x^2*y;
> gl2:=-abl(y,t)+B*x-x^2*y;
```

$$gl1 := -\left(\frac{\partial}{\partial t}x\right) - \varepsilon^2\left(\frac{\partial}{\partial t2}x\right) + 1 - (B+1)\,x + x^2\,y$$

$$gl2 := -\left(\frac{\partial}{\partial t}y\right) - \varepsilon^2\left(\frac{\partial}{\partial t2}y\right) + B\,x - x^2\,y$$

determine the critical eigenvector

```
defxyl:=[x=A+x0*exp(I*omega*t),y=B/A+y0*exp(I*omega*t)];
> expand(subs(defxyl,gl1)):
> coeff(%,exp(I*omega*t));
> defy0:=y0=solve(%,y0);
```

$$defxyl := [x = 1 + x0\,\mathbf{e}^{(I\,\omega\,t)},\, y = B + y0\,\mathbf{e}^{(I\,\omega\,t)}]$$

$$-I\,x0\,\omega + B\,x0 - x0 + y0$$

$$defy0 := y0 = I\,x0\,\omega - B\,x0 + x0$$

```
> expand(subs(defxyl,gl2));
> subs(defy0,%):
> lin2:=factor(coeff(%,exp(I*omega*t)));
```

$$-I\,y0\,\omega\,\mathbf{e}^{(I\,\omega\,t)} - B\,x0\,\mathbf{e}^{(I\,\omega\,t)} - y0\,\mathbf{e}^{(I\,\omega\,t)} - 2\,x0\,(\mathbf{e}^{(I\,\omega\,t)})^2\,y0 - x0^2\,(\mathbf{e}^{(I\,\omega\,t)})^2\,B - x0^2\,(\mathbf{e}^{(I\,\omega\,t)})^3\,y0$$

$$lin2 := (-1 + I\,\omega\,B - 2\,I\,\omega + \omega^2)\,x0$$

```
> defom:=omega=solve(evalc(Re(lin2)),omega)[1];
> defBc:=Bc=solve(subs(defom,lin2),B);
> subs([B=Bc],defy0);
> defyp:=yp=expand(subs([defBc,defom,x0=1],rhs(%)));
> defyps:=yps=subs(I=-I,rhs(defyp));
```
yp is the y-component of the right-eigenvector

$$defom := \omega = 1$$

$$defBc := Bc = 2$$

$$y0 = I\,x0\,\omega - Bc\,x0 + x0$$

$$defyp := yp = -1 + I$$

$$defyps := yps = -1 - I$$

```
> defB:=B=Bc+B2*epsilon^2;
> defypp:=ypp=factor(subs(defom,A^2/(I*omega+A^2)));
```
ypp is the y-component of the left-eigenvector

```
> defypps:=ypps=factor(subs(defom,A^2/(-I*omega+A^2)));
```

$$defB := B = Bc + B2\,\varepsilon^2$$

$$defypp := ypp = \frac{1}{2} - \frac{1}{2}I$$

$$defypps := ypps = \frac{1}{2} + \frac{1}{2}I$$

expansion of x and y

```
> defx:=x=A+epsilon*(A1*exp(I*omega*t)+A1s*exp(-I*omega*t))+epsilon^
  2*((A21+A22)*exp(I*omega*t)+(A21s+A22s)*exp(-I*omega*t)+B21+B21s+(
  C21+C22)*exp(2*I*omega*t)+(C21s+C22s)*exp(-2*I*omega*t))+epsilon^3
  *((A31+A32)*exp(I*omega*t)+(A31s+A32s)*exp(-I*omega*t)+B31+(C31+C3
  2)*exp(2*I*omega*t)+(C31s+C32s)*exp(-2*I*omega*t)+(D31+D32)*exp(3*
```

```
I*omega*t)+(D31s+D32s)*exp(-3*I*omega*t));
> defy:=y=B/A+epsilon*(yp*A1*exp(I*omega*t)+yps*A1s*exp(-I*omega*t))
  +epsilon^2*((yp*A21+yps*A22)*exp(I*omega*t)+(yps*A21s+yp*A22s)*exp
  (-I*omega*t)+yp*B21+yps*B21s+(C21*yp+C22*yps)*exp(2*I*omega*t)+(yp
  s*C21s+yp*C22s)*exp(-2*I*omega*t))+epsilon^3*((yp*A31+yps*A32)*exp
  (I*omega*t)+(yps*A31s+yp*A32s)*exp(-I*omega*t)+yp*B31+yps*B31s+(yp
  *C31+yps*C32)*exp(2*I*omega*t)+(yps*C31s+yp*C32s)*exp(-2*I*omega*t
  )+(yp*D31+yps*D32)*exp(3*I*omega*t)+(yps*D31s+yp*D32s)*exp(-3*I*om
  ega*t));
```

$$defx := x = 1 + \varepsilon\,(A1\,\mathbf{e}^{(I\,\omega\,t)} + A1s\,\mathbf{e}^{(-I\,\omega\,t)}) + \varepsilon^2\,((A21 + A22)\,\mathbf{e}^{(I\,\omega\,t)} + (A21s + A22s)\,\mathbf{e}^{(-I\,\omega\,t)} + B21 + B21s + (C21 + C22)\,\mathbf{e}^{(2\,I\,\omega\,t)} + (C21s + C22s)\,\mathbf{e}^{(-2\,I\,\omega\,t)}) + \varepsilon^3\,((A31 + A32)\,\mathbf{e}^{(I\,\omega\,t)} + (A31s + A32s)\,\mathbf{e}^{(-I\,\omega\,t)} + B31 + (C31 + C32)\,\mathbf{e}^{(2\,I\,\omega\,t)} + (C31s + C32s)\,\mathbf{e}^{(-2\,I\,\omega\,t)} + (\mathrm{D31} + \mathrm{D32})\,\mathbf{e}^{(3\,I\,\omega\,t)} + (D31s + D32s)\,\mathbf{e}^{(-3\,I\,\omega\,t)})$$

$$defy := y = B + \varepsilon\,(yp\,A1\,\mathbf{e}^{(I\,\omega\,t)} + yps\,A1s\,\mathbf{e}^{(-I\,\omega\,t)}) + \varepsilon^2\,((yp\,A21 + yps\,A22)\,\mathbf{e}^{(I\,\omega\,t)} + (yps\,A21s + yp\,A22s)\,\mathbf{e}^{(-I\,\omega\,t)} + yp\,B21 + yps\,B21s + (C21\,yp + C22\,yps)\,\mathbf{e}^{(2\,I\,\omega\,t)} + (yps\,C21s + yp\,C22s)\,\mathbf{e}^{(-2\,I\,\omega\,t)}) + \varepsilon^3\,((yp\,A31 + yps\,A32)\,\mathbf{e}^{(I\,\omega\,t)} + (yps\,A31s + yp\,A32s)\,\mathbf{e}^{(-I\,\omega\,t)} + yp\,B31 + yps\,B31s + (yp\,C31 + yps\,C32)\,\mathbf{e}^{(2\,I\,\omega\,t)} + (yps\,C31s + yp\,C32s)\,\mathbf{e}^{(-2\,I\,\omega\,t)} + (yp\,\mathrm{D31} + yps\,\mathrm{D32})\,\mathbf{e}^{(3\,I\,\omega\,t)} + (yps\,D31s + yp\,D32s)\,\mathbf{e}^{(-3\,I\,\omega\,t)})$$

```
>
> subs([defx,defy,defB],gl1):
> subs([defyp,defyps,defB],%):subs([defBc],%):
> gl1e:=collect(expand(convert(taylor(%,epsilon=0,4),polynom)),[epsi
  lon,exp(I*omega*t)]):
>
>
> subs([defx,defy,defB],gl2):
> subs([defyp,defyps,defB],%):subs([defBc],%):
> gl2e:=collect(expand(convert(taylor(%,epsilon=0,4),polynom)),[epsi
  lon,exp(I*omega*t)]):
```

check whether eigenvector correct

```
> collect(expand(coeff(gl1e,epsilon,1)),exp(I*omega*t)):
> expand(subs([defBc,defom],%));
```

$$0$$

```
> collect(expand(coeff(gl2e,epsilon,1)),exp(I*omega*t)):
> expand(subs([defBc,defom],%));
```

$$0$$

check whether left eigenvector is correct

```
> linop1:=collect(coeff(expand(subs(defxyl,gl1)),exp(I*omega*t)),[x0
  ,y0]);
> linop2:=collect(coeff(expand(subs(defxyl,gl2)),exp(I*omega*t)),[x0
  ,y0]);
> collect(subs([defom,defB,defypp,epsilon=0],linop1+ypp*linop2),[x0,
```

```
y0],factor);
> subs(defBc,%);
```

$$linop1 := (-I\,\omega + B - 1)\,x0 + y0$$

$$linop2 := -B\,x0 + (-I\,\omega - 1)\,y0$$

$$\left(\frac{1}{2} + \frac{1}{2}I\right)(-2 + Bc + B2\,\varepsilon^2)\,x0$$

$$\left(\frac{1}{2} + \frac{1}{2}I\right)B2\,\varepsilon^2\,x0$$

```
> gl1e2:=collect(expand(coeff(gl1e,epsilon,2)),exp(I*omega*t));
```

$$gl1e2 := (2\,I\,A1^2 - 2\,I\,\omega\,C22 - I\,C22 - 2\,I\,\omega\,C21 + I\,C21)\,(\mathbf{e}^{(I\,\omega\,t)})^2 + (I\,A21 - I\,A22 - I\,\omega\,A22 - I\,\omega\,A21)\,\mathbf{e}^{(I\,\omega\,t)} - I\,B21s + I\,B21 + \frac{I\,A22s + I\,\omega\,A22s - I\,A21s + I\,\omega\,A21s}{\mathbf{e}^{(I\,\omega\,t)}} + \frac{I\,C22s - 2\,I\,A1s^2 - I\,C21s + 2\,I\,\omega\,C22s + 2\,I\,\omega\,C21s}{(\mathbf{e}^{(I\,\omega\,t)})^2}$$

```
>
> gl1e20:=coeff(gl1e2,exp(I*omega*t),0);
> gl1e21:=coeff(gl1e2,exp(I*omega*t),1);
> gl1e22:=coeff(gl1e2,exp(I*omega*t),2);
> gl1e2m1:=coeff(gl1e2,exp(I*omega*t),-1);
> gl1e2m2:=coeff(gl1e2,exp(I*omega*t),-2);
```

$$gl1e20 := -I\,B21s + I\,B21$$

$$gl1e21 := I\,A21 - I\,A22 - I\,\omega\,A22 - I\,\omega\,A21$$

$$gl1e22 := 2\,I\,A1^2 - 2\,I\,\omega\,C22 - I\,C22 - 2\,I\,\omega\,C21 + I\,C21$$

$$gl1e2m1 := I\,A22s + I\,\omega\,A22s - I\,A21s + I\,\omega\,A21s$$

$$gl1e2m2 := I\,C22s - 2\,I\,A1s^2 - I\,C21s + 2\,I\,\omega\,C22s + 2\,I\,\omega\,C21s$$

```
> gl2e2:=collect(expand(coeff(gl2e,epsilon,2)),exp(I*omega*t));
```

$$gl2e2 := (-2\,\omega\,C22 + 2\,\omega\,C21 + 2\,I\,\omega\,C22 + 2\,I\,\omega\,C21 - 2\,I\,A1^2 - C22 + I\,C22 - C21 - I\,C21)\,(\mathbf{e}^{(I\,\omega\,t)})^2 + (\omega\,A21 - A22 + I\,\omega\,A22 - I\,A21 + I\,\omega\,A21 - A21 + I\,A22 - \omega\,A22)\,\mathbf{e}^{(I\,\omega\,t)} - I\,B21 - B21s + I\,B21s - B21 + \frac{-A21s - I\,\omega\,A21s + I\,A21s - A22s - I\,\omega\,A22s - I\,A22s - \omega\,A22s + \omega\,A21s}{\mathbf{e}^{(I\,\omega\,t)}} + \frac{-I\,C22s - 2\,\omega\,C22s + I\,C21s - 2\,I\,\omega\,C22s + 2\,I\,A1s^2 - C21s - C22s + 2\,\omega\,C21s - 2\,I\,\omega\,C21s}{(\mathbf{e}^{(I\,\omega\,t)})^2}$$

```
> gl2e20:=coeff(gl2e2,exp(I*omega*t),0);
> gl2e21:=coeff(gl2e2,exp(I*omega*t),1);
> gl2e22:=coeff(gl2e2,exp(I*omega*t),2);gl2e2m1:=coeff(gl2e2,exp(I*o
  mega*t),-1);
> gl2e2m2:=coeff(gl2e2,exp(I*omega*t),-2);
```

$$gl2e20 := I\,B21s - I\,B21 - B21s - B21$$
$$gl2e21 := \omega\,A21 - A22 + I\,\omega\,A22 - I\,A21 + I\,\omega\,A21 - A21 + I\,A22 - \omega\,A22$$
$$gl2e22 := -2\,\omega\,C22 + 2\,\omega\,C21 + 2\,I\,\omega\,C22 + 2\,I\,\omega\,C21 - 2\,I\,A1^2 - C22 + I\,C22 - C21 - I\,C21$$
$$gl2e2m1 := -A21s - I\,\omega\,A21s + I\,A21s - A22s - I\,\omega\,A22s - I\,A22s - \omega\,A22s + \omega\,A21s$$
$$gl2e2m2 := -I\,C22s - 2\,\omega\,C22s + I\,C21s - 2\,I\,\omega\,C22s + 2\,I\,A1s^2 - C21s - C22s + 2\,\omega\,C21s - 2\,I\,\omega\,C21s$$

```
> defA22:=A22=subs(defom,solve(gl2e21,A22));
> defA22s:=A22s=subs(defom,solve(gl2e2m1,A22s));
```

$$defA22 := A22 = 0$$
$$defA22s := A22s = 0$$

```
> defB2:=solve({gl1e20,gl2e20},{B21,B21s});
```

$$defB2 := \{B21 = 0, B21s = 0\}$$

```
> defC2:=solve({gl1e22,gl2e22},{C21,C22});
> defC2s:=solve({gl1e2m2,gl2e2m2},{C21s,C22s});
```

$$defC2 := \{C21 = \frac{A1^2}{2\,\omega - 1}, C22 = \frac{A1^2}{2\,\omega + 1}\}$$
$$defC2s := \{C21s = \frac{A1s^2}{2\,\omega - 1}, C22s = \frac{A1s^2}{2\,\omega + 1}\}$$

test whether equations at quadratic order solved correctly

```
> subs(defom,normal(subs([defA22,defA22s,defB2[1],defB2[2],defC2[1],
  defC2[2],defC2s[1],defC2s[2]],gl2e2)));
> subs(defom,normal(subs([defA22,defA22s,defB2[1],defB2[2],defC2[1],
  defC2[2],defC2s[1],defC2s[2]],gl1e2)));
```

$$0$$
$$0$$

```
>
> gl1e3:=collect(expand(coeff(gl1e,epsilon,3)),exp(I*omega*t));
```

$$\begin{aligned}
gl1e3 := {}& (-A1^3 - 3\,I\,\omega\,\mathrm{D31} - I\,\mathrm{D32} + I\,\mathrm{D31} - 3\,I\,\omega\,\mathrm{D32} + 4\,I\,A1\,C21 + I\,A1^3)\,(\mathbf{e}^{(I\,\omega\,t)})^3 \\
& + (I\,C31 + 4\,I\,A1\,A21 - I\,C32 - 2\,I\,\omega\,C32 - 2\,I\,\omega\,C31)\,(\mathbf{e}^{(I\,\omega\,t)})^2 + \Bigg(I\,A1^2\,A1s + I\,A31 - I\,A32 \\
& + 4\,I\,A1\,B21 + B2\,A1 - 4\,I\,A1s\,C22 - I\,\omega\,A32 - \left(\frac{\partial}{\partial t2}A1\right) - I\,\omega\,A31 - 3\,A1^2\,A1s\Bigg)\,\mathbf{e}^{(I\,\omega\,t)} + I\,B31 \\
& - 4\,I\,A1s\,A22 - B31s + 4\,I\,A1\,A22s - I\,B31s + \Bigg(-\left(\frac{\partial}{\partial t2}A1s\right) + B2\,A1s + I\,\omega\,A32s - I\,A1\,A1s^2 \\
& - I\,A31s + I\,\omega\,A31s - 3\,A1\,A1s^2 + 4\,I\,A1\,C22s + I\,A32s - 4\,I\,A1s\,B21s\Bigg) \Big/ \mathbf{e}^{(I\,\omega\,t)} \\
& + \frac{-I\,C31s + I\,C32s + 2\,I\,\omega\,C31s - 4\,I\,A1s\,A21s + 2\,I\,\omega\,C32s}{(\mathbf{e}^{(I\,\omega\,t)})^2}
\end{aligned}$$

$$+\frac{-A1s^3 - I\,A1s^3 - I\,D31s + 3\,I\,\omega\,D31s + I\,D32s - 4\,I\,A1s\,C21s + 3\,I\,\omega\,D32s}{\left(e^{(I\,\omega\,t)}\right)^3}$$

```
> gl2e3:=collect(expand(coeff(gl2e,epsilon,3)),exp(I*omega*t));
```

$$gl2e3 := (-3\,\omega\,\mathrm{D32} - \mathrm{D32} - \mathrm{D31} + I\,\mathrm{D32} + 3\,I\,\omega\,\mathrm{D31} + A1^3 - 4\,I\,A1\,C21 - I\,\mathrm{D31} + 3\,\omega\,\mathrm{D31}$$
$$+ 3\,I\,\omega\,\mathrm{D32} - I\,A1^3)\left(e^{(I\,\omega\,t)}\right)^3 +$$
$$(-4\,I\,A1\,A21 + I\,C32 + 2\,I\,\omega\,C31 + 2\,\omega\,C31 - 2\,\omega\,C32 - C32 + 2\,I\,\omega\,C32 - C31 - I\,C31)\left(e^{(I\,\omega\,t)}\right)^2$$
$$+\left(-I\,A31 + 4\,I\,A1s\,C22 - 4\,I\,A1\,B21 + 3\,A1^2\,A1s + I\,A32 + \left(\frac{\partial}{\partial t2}A1\right) - A31 + \omega\,A31 - A32 - \omega\,A32\right.$$
$$\left. + I\,\omega\,A32 - I\,A1^2\,A1s + I\,\omega\,A31 - B2\,A1 - I\left(\frac{\partial}{\partial t2}A1\right)\right)e^{(I\,\omega\,t)} + B31s + I\,B31s - B31 - I\,B31$$
$$+ 4\,I\,A1s\,A22 - 4\,I\,A1\,A22s + \left(-I\,A32s - \omega\,A32s - I\,\omega\,A32s + \left(\frac{\partial}{\partial t2}A1s\right) + 3\,A1\,A1s^2\right.$$
$$+ 4\,I\,A1s\,B21s + I\left(\frac{\partial}{\partial t2}A1s\right) - 4\,I\,A1\,C22s + I\,A1\,A1s^2 - B2\,A1s - A32s - I\,\omega\,A31s - A31s$$
$$\left. + \omega\,A31s + I\,A31s\right)\Big/ e^{(I\,\omega\,t)} + ($$
$$-2\,I\,\omega\,C31s - C32s - I\,C32s - 2\,I\,\omega\,C32s + I\,C31s - 2\,\omega\,C32s - C31s + 4\,I\,A1s\,A21s + 2\,\omega\,C31s)/$$
$$\left(e^{(I\,\omega\,t)}\right)^2 + (A1s^3 - D31s + I\,A1s^3 - D32s - 3\,I\,\omega\,D32s + 3\,\omega\,D31s - 3\,I\,\omega\,D31s - 3\,\omega\,D32s$$
$$+ 4\,I\,A1s\,C21s + I\,D31s - I\,D32s)\Big/\left(e^{(I\,\omega\,t)}\right)^3$$

```
> subs([defB2[1],defB2[2],defC2[1],defC2[2],defC2s[1],defC2s[2]],gl1e3):
> collect(expand(%),exp(I*omega*t)):
> gl1e31:=collect(coeff(%,exp(I*omega*t),1),[A1,A1s],distributed,factor);
>
```

$$gl1e31 :=$$
$$-I\left(A32 + \omega\,A31 - A31 - I\left(\frac{\partial}{\partial t2}A1\right) + \omega\,A32\right) + B2\,A1 - \frac{\left(\frac{3}{5} - \frac{1}{5}\,I\right)(10\,\omega + 3 + 6\,I)\,A1^2\,A1s}{2\,\omega + 1}$$

```
> subs([defB2[1],defB2[2],defC2[1],defC2[2],defC2s[1],defC2s[2]],gl2e3):
> collect(expand(%),exp(I*omega*t)):
> gl2e31:=collect(coeff(%,exp(I*omega*t),1),[A1,A1s],distributed,factor);
>
```

$$gl2e31 :=$$

$$(1+I)\left(\omega\, A31 - A31 - I\left(\frac{\partial}{\partial t2} A1\right) + I\, A32 + I\,\omega\, A32\right) - B2\, A1 + \frac{\left(\frac{3}{5} - \frac{1}{5} I\right)(10\,\omega + 3 + 6\, I)\, A1^2\, A1s}{2\,\omega + 1}$$

project on left-zero-eigenvector to yield the solvability condition, i.e. the amplitude equation

```
> normal(gl1e31+ypp*gl2e31):
> subs(defypp,%):
> subs([defom,defBc],%):
> normal(evalc(collect(%,[A1,A1s],distributed,factor))):
> cgl:=diff(A1,t2)=solve(%,diff(A1,t2)):
> collect(%,[A1,A1s],factor);
```

$$\frac{\partial}{\partial t2} A1 = -\left(\frac{3}{2} + \frac{1}{6} I\right) A1s\, A1^2 + \frac{1}{2} B2\, A1$$

Test whether equations are actually satisfied to this order by the solutions determined

```
> gl1e31e:=subs(cgl,gl1e31);
```

$$gl1e31e :=$$

$$-I\left(A32 + \omega\, A31 - A31 + \left(\frac{1}{12} + \frac{1}{12} I\right) A1\,(-3\, B2 + 8\, A1\, A1s - 3\, I\, B2 + 10\, I\, A1\, A1s) + \omega\, A32\right)$$
$$+ B2\, A1 - \frac{\left(\frac{3}{5} - \frac{1}{5} I\right)(10\,\omega + 3 + 6\, I)\, A1^2\, A1s}{2\,\omega + 1}$$

```
> gl2e31e:=subs(cgl,gl2e31);
```

$$gl2e31e :=$$

$$(1+I)\left(\omega\, A31 - A31 + \left(\frac{1}{12} + \frac{1}{12} I\right) A1\,(-3\, B2 + 8\, A1\, A1s - 3\, I\, B2 + 10\, I\, A1\, A1s) + I\, A32 + I\,\omega\, A32\right)$$
$$- B2\, A1 + \frac{\left(\frac{3}{5} - \frac{1}{5} I\right)(10\,\omega + 3 + 6\, I)\, A1^2\, A1s}{2\,\omega + 1}$$

```
> defA32:=A32=collect(subs(defom,solve(gl2e31e,A32)),[A1,A1s],distributed,factor);
```

$$defA32 := A32 = -\left(\frac{1}{12} - \frac{3}{4} I\right) A1s\, A1^2 - \frac{1}{4} I\, B2\, A1$$

```
> normal(subs(defom,subs(defA32,gl1e31e)));
```

$$0$$

```
> collect(subs([defA22,defA22s,defB2[1],defB2[2],defC2[1],defC2[2],defC2s[1],defC2s[2],cgl,defA32,defBc],gl1e),[epsilon,exp(I*omega*t)],factor):subs(defom,%);test:=coeff(%,exp(I*A*t));
```

$$\Bigg(-I(-I\, A1^3 + 2\,\mathrm{D31} + 4\,\mathrm{D32} - 5\, A1^3)\left(\mathbf{e}^{(I\,t)}\right)^3 - I(C31 - 4\, A1\, A21 + 3\, C32)\left(\mathbf{e}^{(I\,t)}\right)^2$$

$$-\left(\frac{1}{2}+\frac{1}{2}I\right)(2\,B31s-B31-I\,B31)$$

$$+\frac{\frac{1}{3}I\left(3\,I\left(\frac{\partial}{\partial t2}A1s\right)-3\,I\,B2\,A1s+6\,A32s+A1\,A1s^2+9\,I\,A1\,A1s^2\right)}{\mathbf{e}^{(I\,t)}}$$

$$\left.+\frac{I\,(C31s+3\,C32s-4\,A1s\,A21s)}{\left(\mathbf{e}^{(I\,t)}\right)^2}+\frac{I\,(I\,A1s^3-5\,A1s^3+2\,D31s+4\,D32s)}{\left(\mathbf{e}^{(I\,t)}\right)^3}\right)\varepsilon^3$$

$$test := 0$$

```
> collect(subs([defA22,defA22s,defB2[1],defB2[2],defC2[1],defC2[2],d
  efC2s[1],defC2s[2],cgl,defA32,defBc],gl2e),[epsilon,exp(I*omega*t)
  ],factor):subs(defom,%);test:=coeff(%,exp(I*A*t));
```

$$\left(-(1-I)\,(-3\,A1^3+4\,\mathrm{D32}-2\,I\,\mathrm{D31}+2\,I\,A1^3)\left(\mathbf{e}^{(I\,t)}\right)^3\right.$$

$$-(1-I)\,(-2\,A1\,A21+3\,C32-I\,C31+2\,I\,A1\,A21)\left(\mathbf{e}^{(I\,t)}\right)^2+(1+I)\,(-B31+B31s)$$

$$-\frac{\left(\frac{1}{6}-\frac{1}{6}I\right)\left(-10\,A1\,A1s^2+3\,B2\,A1s-8\,I\,A1\,A1s^2+3\,I\,B2\,A1s+12\,I\,A32s-6\,I\left(\frac{\partial}{\partial t2}A1s\right)\right)}{\mathbf{e}^{(I\,t)}}$$

$$-\frac{(1-I)\,(3\,I\,C32s-C31s-2\,I\,A1s\,A21s+2\,A1s\,A21s)}{\left(\mathbf{e}^{(I\,t)}\right)^2}$$

$$\left.-\frac{(1-I)\,(-3\,I\,A1s^3-2\,D31s+4\,I\,D32s+2\,A1s^3)}{\left(\mathbf{e}^{(I\,t)}\right)^3}\right)\varepsilon^3$$

$$test := 0$$

```
>
```