ESAM 438
Interdisciplinary Nonlinear Dynamics

Fall 2000

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Problem Set 3

For Discussion Section October 18

1. **Perturbed Transcritical Bifurcation**

   To *unfold* the transcritical bifurcation consider adding a perturbation h to obtain

   $$\partial_t x = \mu x - x^2 + h. \tag{1}$$

   Determine all possible bifurcation diagrams that are obtained as μ or h are varied, respectively. Try to sketch the solution surface $x = x(\mu, h)$. By projecting onto the (μ, h)-plane, determine the complete phase diagram, which shows how many solutions are obtained for any combination of the two parameters x and h.

2. **Perturbed Pitch-Fork Bifurcation**

   In class we considered the equation

   $$\partial_t x = \mu x - x^3 + h \tag{2}$$

   as the general form of perturbing the pitch-fork bifurcation. Often, instead the equation

   $$\partial_t y = \nu y - y^3 + \alpha + \beta y^2 \tag{3}$$

   is considered. It makes the fact more apparent that a special feature of the pitch-fork bifurcation is the missing of **all** even terms in x.

   (a) Introduce a suitable variable transformation form relating y and x as well as ν and μ that brings (3) into the form (2). What is the connection between the coefficients ν, α, β, and μ and h?

   (b) Use (3) to show that among the perturbations of the pitch-fork bifurcation are also diagrams of the form shown below.
   Can such a bifurcation diagram also be obtained in (2)? If so, how do the parameters μ and h have to be varied to get it?

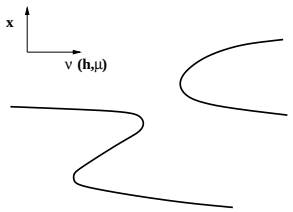

Figure 1: One possible perturbation of the pitch-fork bifurcation

3. **Non-generic Scaling for Period**

   Strogatz: 4.3.10.

4. **Superconducting Josephson Junction**

   Read Ch.4.6 in Strogatz and do the problems: 4.6.1, 4.6.2, 4.6.3.