Interdisciplinary Nonlinear Dynamics 438-1,2,3

July 30, 2000

This course will constitute a core element of the new NSF/IGERT program *Dynamics of Complex Systems in Science and Engineering*. It will start as a lecture-based course in fall. In the winter and spring quarter it will evolve into a projects-based course.

I. Lecture Course in Fall Quarter

The class will meet twice a week. Lectures will often include numerical demonstrations. There will be regular homework assignments, which will include analytical and numerical work. The latter will be done mostly using Matlab. The homework will be graded by teaching assistants, who will also lead discussion/tutorial sections. The teaching assistants will be senior students in the NSF/IGERT program *Dynamics of Complex Systems in Science and Engineering*.

Text Book: S. Strogatz, *Nonlinear Dynamics and Chaos,* Addison-Wesley, 1994.

Course Outline:

1. Introduction
   Basic properties of differential equations: linear *vs.* nonlinear.
   Reading: Ch. 1, 2.0-2.2

2. One-dimensional Flow
   Fixed points and stability, linear stability analysis, impossibility of oscillations, numerical methods.
   Reading: Ch. 2.3-2.8

3. Bifurcations
   Implicit-function theorem, saddle-node bifurcation, transcritical bifurcation, laser threshold.
   Reading: Ch. 3.0-3.3

4. Bifurcations
   Pitchfork bifurcation, imperfect bifurcations and catastrophes, insect outbreak.
   Reading: Ch. 3.4-3.7

5. Flows on the Circle
   Uniform and nonuniform oscillator, bifurcation, overdamped pendulum, superconducting Josephson junction.
   Reading: Ch. 4

6. Two-dimensional Flows
   Classification of linear systems.
   Reading: Ch. 5

7. Phase Plane
   Phase portraits, fixed points and linearization, attractors and basin of attraction, pendulum.
   Reading: Ch. 6

8. Limit Cycles
   Ruling out oscillations: Lyapunov functions, Poincare-Bendixson theorem: no chaos in two dimensions.
   Reading: Ch. 7.0-7.3, 7.5

9. Weakly Nonlinear Oscillators
   Regular perturbation theory and its break-down, two-timing, averaged equations.
   Reading: Ch. 7.6

10. Bifurcations in 2 Dimensions
    Hopf bifurcation. Complex amplitude equation in the vicinity of the bifurcation.
    Reading: Ch. 8

11. Bifurcations in 2 Dimensions
    Pitchfork bifurcation: separation of time scales. Stable, unstable, and center manifold. Center manifold reduction.
    Reading: Ch. 8

12. Bifurcations and Partial Differential Equations
    Pattern formation: Rayleigh-Benard convection and Swift-Hohenberg model. Neutral curve and amplitude equation. Modulation theory and slow spatial scales: Ginzburg-Landau equation.
    Reading: tba

13. Large-scale Dynamics of Patterns
    Phase dynamics in the Ginzburg-Landau equation. Stability of steady patterns: Eckhaus instability.
    Reading: tba

14. Lorenz Equations
    Model for convection, water wheel. Dissipation and phase-space volume contraction. Fixed points and limit cycles.
    Reading: Ch. 9.0-9.2

15. Lorenz Attractor
    Strange attractor, sensitive dependence on initial conditions, Lyapunov exponents, Lorenz map.
    Reading: Ch. 9.3-9.5

16. One-dimensional Maps
    Logistic map. Fixed points and cobwebs, periodic points and iterates of maps. Numerical results: period-doubling cascade, periodic windows, intermittency. Lyapunov exponent.
    Reading: Ch. 10.0-10.4

17. One-dimensional Maps
    Scaling of the period-doubling cascade. Renormalization and universality. One-dimensional maps and experiments.
    Reading: Ch. 10.5-10.7

18. Strange Attractors
    Cantor set and fractal dimension. Baker's map, stretching and folding. Henon map.
    Reading: Ch. 11

19. Diagnostic Tools
    Attractor reconstruction. Rössler attractor. Poincare section. Return map. Power spectrum. Comparison with experiments.
    Reading: Ch. 12

20. Outlook
    Excitable systems, localized pulses. Spatio-temporal chaos.

II. Project-Based Course in Winter and Spring Quarters

In the Winter and Spring quarters the students will split into 2 or 3 project-focussed teams comprised of approximately 5 students each and supervised by 2 faculty members with complementary expertise and perspectives. A non-exhaustive list of examples of research areas from which the themes for the projects may be chosen are listed below. The students in a given team will focus on one theme, with each student pursuing his or her own project as part of the larger group effort; this ensures that each student learns all aspects of a specific area, while still gaining valuable team-work experience. For example, some students may set out to reproduce theoretical results of research papers in the literature, with the ultimate goal of adding their own contributions to the topic, while other students may read and critique experimental papers, before going on to do laboratory work related to the team research theme. Progress in nonlinear science rests on the close interplay among theory, experiment, and computation. This will be reflected in the course by including a strong computational component in each student's experimental and/or theoretical project.

Background material that is relevant to all students in a team will be addressed in weekly group meetings; the role of discussion leader will rotate among the students and faculty in the group. In these meetings the students will also briefly report on progress in their work and on possible difficulties. This gives the students ample opportunity to practice their presentation skills and to incorporate the feedback they receive from their fellow students and the mentoring faculty in preparing their final project presentation.

In addition to the weekly meetings of the teams, there will be a weekly meeting of all students and faculty participating in the course. There will be one primary faculty member who will coordinate this aspect of the course, although responsibility for the common lectures will cycle between the groups. Initially, for instance, the faculty may take a lead in the class meetings by giving some overview lectures of their team's research area. As the students progress in their own work, they will start presenting the highlights in the style of a journal-club presentation. These presentations will provide the background for the capstone of the course: the final presentations by the students in each of the teams. A class discussion following each presentation will give students valuable feedback that they can take into account in preparing their final written project report.

Non-exhaustive List of Themes:

1. **Pattern Formation:**
   Students will investigate pattern formation in physical, chemical, or biological systems. This could, for instance, be wave patterns in periodically forced systems (e.g. surface waves on liquids or granular media) or waves in excitable media (e.g. in the heart, neurons, EEG-activity). The investigations will include analytical and computational work as well as experiments in the lab of Prof. Umbanhowar.

2. **Neurocontrol of Biomorphic Systems:**
   The projects will combine concepts and tools of theoretical neurocomputing and nonlinear adaptive control in nonlinear limb dynamics, and apply them to the control of a biomimetic robotic system in the lab of Prof. Mussa-Ivaldi.

3. **Solitons and Solitary Structures**:
   The projects will focus on localized structures in optical and other physical or chemical systems. Students will set up experiments on soliton propagation in optical fibers in the lab of Prof. Kumar. Comparison with theory will involve numerical simulations of the corresponding analytical models of nonlinear pulse propagation. The behavior of nonlinear waves in optical systems will be contrasted with their behavior in other physical and biological systems.

4. **From Low- to High-Dimensional Chaos:**
   Each project will require mastery of the same basic set of techniques of the theory of low-dimensional chaotic systems, including computation and graphical visualization. Applications may be as varied as neurophysiological experiments, analysis and experiments of mixing in fluids and granular material, or numerical simulations and analysis of spatiotemporal chaos.