

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \epsilon A_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \epsilon^2 \left\{ A_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + B_2 \begin{pmatrix} 1 \\ -b \end{pmatrix} \right\} + \\ + \epsilon^3 \left\{ A_3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + B_3 \begin{pmatrix} 1 \\ -b \end{pmatrix} \right\} + \dots$$

Center manifold reduction:

slow:  $\underline{e}_1$  in  $y$ -direction

$\Rightarrow$  fast direction eliminated in favor of slow variable

$$x = h(y)$$

$$\dot{x} = h'(y) \dot{y} = -x + \mu y - y^3 - axy^2 = h'(y) b x$$

$$\Rightarrow -h + \mu y - y^3 - ahy^2 = h'(y) b h$$

due to reflection symmetry  $x \rightarrow -x$   
 $y \rightarrow -y$

Expect pitch-fork bifurcation:

$$\dot{y} = \lambda y - y^3 + \dots$$

with  $\lambda = \sigma(\mu)$  and therefore

$$\mu = \sigma(y^2)$$

$x$  and  $y$  have same symmetry

$\Rightarrow h$  is odd in  $y$

$h$  strictly nonlinear in  $y$  and  $\mu$