Interdisciplinary Nonlinear Dynamics (438)

Fall 2001

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Problem Set 3

For Discussion Section October 19

1. **Hysteresis and Jumps**

   a) Study the following model numerically,

$$\frac{du}{dt} = e^{0.3u}\sin 2u + e^{-0.1u} + A(t) \tag{1}$$

$$A(t) = \sin(\omega t). \tag{2}$$

   Use as initial condition $u(t=0)=0$ and run the solution to $t_{max}=200$. Choose $\omega=0.05$ to mimic a slow change of the "forcing" A. How does the time-dependence of the solution change qualitatively when you increase the amplitude A in steps from $A=1$ to $A=23$? Make sure that your solution is numerically resolved. Plot in the same graph $A(t)$ and use it to identify ranges of A for which the solution exhibits hysteresis.

   b) Interpret the result in terms of the bifurcations that the solutions to (1) undergo when $A(t)$ is replaced by a constant A_0 and A_0 is scanned over the range covered by $A(t)$ in your simulation.

2. **Perturbed Transcritical Bifurcation**

   To *unfold* the transcritical bifurcation consider adding a perturbation h to obtain

$$\partial_t x = \mu x - x^2 + h. \tag{3}$$

   Determine all possible bifurcation diagrams that are obtained as μ or h are varied, respectively. Try to sketch the solution surface $x = x(\mu, h)$. By projecting onto the (μ, h)-plane, determine the complete phase diagram, which shows how many solutions are obtained for any combination of the two parameters μ and h.

3. **Perturbed Pitch-Fork Bifurcation**

   In class we considered the equation

$$\partial_t x = \mu x - x^3 + h \tag{4}$$

   as the general form of perturbing the pitch-fork bifurcation. Often, instead the equation

$$\partial_t y = \nu y - y^3 + \alpha + \beta y^2 \tag{5}$$

   is considered. It makes the fact more apparent that a special feature of the pitch-fork bifurcation is the missing of **all** even terms in x.

   (a) Introduce a suitable variable transformation form relating y and x as well as ν and μ that brings (5) into the form (4). What is the connection between the coefficients ν, α, β, and μ and h?

   (b) Use (5) to show that among the perturbations of the pitch-fork bifurcation are also diagrams of the form shown below.

   Can such a bifurcation diagram also be obtained in (4)? If so, how do the parameters μ and h have to be varied to get it?

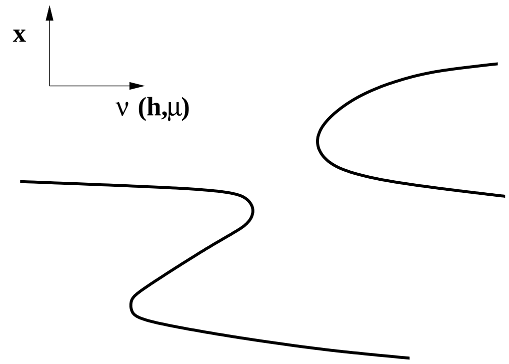

Figure 1: One possible perturbation of the pitch-fork bifurcation

4. **Non-generic Scaling for Period**

   Strogatz: 4.3.10.

5. **Superconducting Josephson Junction**

   Read Ch.4.6 in Strogatz and do the problems: 4.6.1, 4.6.2, 4.6.3.