Interdisciplinary Nonlinear Dynamics (438)
Fall 2001
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Problem Set 5

For Discussion Section November 2

1. **Damped Driven Pendulum**

   Consider a pendulum with friction that is exposed to a constant torque Γ,

   $$ml^2\frac{d^2\theta}{dt^2}+\beta\frac{d\theta}{dt}=-mgl\sin\theta+\Gamma. \tag{1}$$

   (a) Rewrite (1) as a system of first-order equations and determine all fixed points of this system.

   (b) Perform a linear stability analysis of all the fixed points. Discuss what happens physically when the fixed points become unstable! Can you use this to make an educated guess as to what kind of bifurcation the instability leads to?

2. **Relaxation Oscillations**

   Read Ch.7.5 in Strogatz and do problem 7.5.4. Solve also the equations numerically and measure (roughly) the period of the oscillation. Compare the numerically determined period with that from the analytical approximation obtained for $\mu \gg 1$.

3. **Weakly Nonlinear Analysis of Hopf Bifurcation**

   Consider the two coupled equations

   $$\frac{dx}{dt} = \mu x + y + ax^3, \tag{2}$$
   $$\frac{dy}{dt} = -x + \mu y + by^3. \tag{3}$$

   (a) Sketch the nullclines of (2,3) and use them to determine qualitatively the flow as much as that is possible from that information.

   (b) Perform a linear stability analysis of the fixed point $(0,0)$ including the eigenvectors associated with the relevant eigenvalues.

   (c) Solve (2,3) numerically (write your own code or use `pplane.m`[1]) and compare with the weakly nonlinear analysis. Specifically, measure (roughly) the amplitude of the oscillations and their frequency for a suitable range of the control parameter μ.

   (d) Use the eigenvectors as the first-order terms in a weakly nonlinear analysis of the Hopf bifurcation. Depending on a and b, can the bifurcation be supercritical and/or subcritical?

[1] See the class web site for downloading that software.