Interdisciplinary Nonlinear Dynamics (438)

Fall 2001

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Problem Set 6

For Discussion Section November 9

1. Do problem 3c and 3d from previous homework assignment.

2. **An Interesting Dynamical System**[1]

   Consider the dynamical system

$$\frac{dx}{dt} = y, \tag{1}$$

$$\frac{dy}{dt} = \alpha x + \beta y + a x^3 - x^2 y. \tag{2}$$

   In this problem you will study this system analytically and numerically. For the numerical simulations you can use `pplane5`[2] or write your own code. `pplane` has the advantage to provide you right away with a number of graphical options.

   (a) Determine analytically the stability of the fixed point $(x, y) = (0, 0)$ as a function of α and β. Based on this information, what type of behaviors may be expected as α and/or β is varied?

   (b) Consider the case $a = -1$.

      i. Solve (1,2) numerically. What kind of nonlinear behavior do you find if you vary α across the instability identified above for $\beta = -1$?
      For $\alpha = 0$ and $\beta = -1$ identify the center manifold of the attractor numerically by starting from a number of different initial conditions. Increase now α to 0.05, 0.1, and 1. What happens to the center manifold?

      ii. For small α perform analytically a center-manifold reduction of (1,2), i.e. determine a center manifold that describes the dynamics of this system for small amplitudes x, y, and for small values of the bifurcation parameter α. Go to cubic order. Obtain the evolution equation for the dynamics of the system on that manifold. Compare the analytically obtained center manifold and the fixed points on it quantitatively with your numerical results.

[1] This is one of the interesting situations that can arise at a codimension-2 bifurcation, i.e. when two modes go unstable at the same parameter value (cf. Guckenheimer & Holmes, *Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields*, Ch. 7.3)

[2] You may want to set the solver to `ODE23` and set the calculation window to 10 times the display window.