Interdisciplinary Nonlinear Dynamics (438)

Fall 2001

Hermann Riecke

Problem Set 8

For Discussion Section December 7

1. **Poincaré Section and Return Map for Rössler System**

   Consider the Rössler system (cf. Strogatz, ch. 12.3)

$$\frac{dx}{dt} = -y - z, \tag{1}$$

$$\frac{dy}{dt} = x + ay, \tag{2}$$

$$\frac{dz}{dt} = b + z(x - c). \tag{3}$$

   Write a Matlab program to solve (1,2,3) numerically (using simply forward Euler).

   (a) Pick $a = 0.2$ and $b = 0.2$. Scan c through the range $5 < c < 7.5$ and plot the projection of the attractor on the (x, y)-plane. To get a reasonable impression of the attractor you have to run at least to $t = 500$ for each value of the parameters.

   (b) Obtain the Poincaré section (surface of section) of the attractor in the (y, z)-plane for increasing values of x, i.e. plot the points (y_n, z_n) at which the attractor pierces the (y, z)-plane with increasing x. Thus, whenever x changes sign (with x increasing) determine y and z by linear interpolation for the time at which $x = 0$. Plot this discrete sequence of y_n and z_n as a function of n.

   (c) Since the points on the Poincaré section obtained in 1b lie on a line which can be characterized by taking only the y_n-values (or the z_n-values), obtain the return map $y_{n+1} = f(y_n)$ by plotting the points (y_n, y_{n+1}). The return map is similar in spirit to the Lorenz map for the maxima discussed in class. Plot the function f for increasing values of c (with $a = b = 0.2$). This sequence of maps should give you an idea of what the dynamics of the Rössler system is in this regime. How does it compare to that of the logistic map?

2. **Universality of Unimodal Maps**

   From Strogatz do problems 10.6.1 and 10.6.2 (note the introductory paragraph on p. 394).