# Interdisciplinary Nonlinear Dynamics (438)

Fall 2002

Hermann Riecke

Problem Set 3

For Discussion Section October 30

1. **Dynamics Near Fixed Points**

   (a) Calculate the eigenvectors and eigenvalues of

   $$\frac{d\mathbf{x}(t)}{dt} = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \mathbf{x}(t) \tag{1}$$

   and use them to give the general solution to (1).

   (b) Calculate the general solution $\mathbf{x}(t)$ (for general initial condition $\mathbf{x}_0$) of

   $$\frac{d\mathbf{x}(t)}{dt} = \begin{pmatrix} -\lambda - \epsilon & 1 \\ 0 & -\lambda \end{pmatrix} \mathbf{x}(t) \tag{2}$$

   with $\lambda > 0$ and $\epsilon > 0$. Determine from it a general orbit in the phase plane. Sketch the phase portrait of this system.

   Sketch now the phase portrait for a degenerate node, i.e. consider the limit $\epsilon \to 0$ of (2). Again obtain first the general solution and a general orbit.

2. **Phase Portraits**

   Do Strogatz 6.1.4, 6.1.13.

3. **Different Types of Stability**

   (a) Give an example of a two-dimensional system that is Lyapunov stable but not asymptotically nor linearly stable.

   (b) Give an example of a system that is asymptotically stable but not linearly stable.

   (c) Give an example of a system that is asymptotically unstable but not linearly unstable.

4. **Population Growth with Eggs III**

   Consider

   $$\frac{dN}{dt} = -N(t) - N(t)^2 + \alpha N(t-\tau) - \beta N(t-\tau)^2. \tag{3}$$

   To get a better overview of the possible behavior of that equation investigate it analytically.

   (a) Determine all fixed points $N_0$ of the equation and linearize around a non-trivial fixed point $N_0 \neq 0$,

   $$N = N_0 + N_1\, e^{\lambda t}, \tag{4}$$

   to obtain an equation for the growth rate $\lambda$.

   (b) Can this fixed point undergo an instability with respect to a monotonic perturbation, i.e. with a real $\lambda$ going through 0? If so, determine the onset of the instability numerically[1].

[1] You can use `fzero` in matlab.

(c) Can this fixed point undergo an instability with respect to an oscillatory perturbation, i.e. with the real part of the complex $\lambda$ going through 0? If so, determine the onset of the instability numerically. Note that there may occur more than one instability. How can this system support oscillations although the differential equation is only first-order in time?

(d) Do your analytical results match your findings in the numerical study of (3) performed in the previous homework?