ESAM 495 Introduction to Computational Neuroscience

Fall Quarter 2008

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Problem Set 1 Due Tuesday, October 7, 2008

1. **Reversal Potentials**

   (a) Consider a cell with an internal Ca^{2+}-concentration of 10^{-4}mM that is placed in a bath with a Ca^{2+}-concentration of 1.5mM. What is the resulting reversal potential $E_{Ca^{2+}}$ if the temperature is $15°C$ (which is approximately $288K$) ? (Note that $k_B/q = 8.6 \cdot 10^{-5} V/K$).

   (b) Suppose that a squid giant axon features the following concentrations: $[K^+]_{out} = 20mM$, $[K^+]_{in} = 400mM$, $[Na^+]_{out} = 440mM$, $[Na^+]_{in} = 50mM$, $[Cl^-]_{out} = 560mM$, $[Cl^-]_{in} = 50mM$. At $20°C$ find the resting potential V_{rest} if the moibility ratios are given by $P_K : P_{Na} : P_{Cl} = 1 : 0.03 : 0.1$. If during an action potential the ratios change to $P_K : P_{Na} : P_{Cl} = 1 : 15 : 0.1$, compute the maximal voltage V_m during that action potential.

2. **Passive Neuron**

   (a) Consider a neuron modeled by the Hodgkin-Huxley equations in which all voltage-gated channels are blocked. Assume it has a capacitance of $C_M = 1\mu F/cm^2$, a leak resistance of $R_M = 10^4 \Omega cm^2$, and a resting potential of $V_r = -65mV$. What is the reversal potential of the leak current? Find an expression for the voltage $V(t)$, $0 \leq t < \infty$, for this neuron if the initial voltage is $V(0) = V_r$ and a current $I_e = 5\mu A/cm^2$ is injected from $t = 0$ to $t = 30ms$.
   Hint: your solution will consist of two expressions, one valid for $0 \leq t < 30ms$ and one for $30ms \leq t$. Note that the voltage cannot make a jump at any time.

   (b) Consider the differential equation

   $$\frac{dV}{dt} = \lambda V + I_0 \sin \omega t. \tag{1}$$

   i. Analytically find a particular solution $V_p(t)$ of (1) that is periodic in time. What initial condition does it satisfy?

   ii. Solve (1) numerically using the forward Euler method and the backward Euler method for $\lambda = -200$, $I_0 = 10000$, $\omega = 0.1$ with $t_{max} = 20$. Use as initial condition $V_p(t = 0)$ as determined in part 2a. A sample matlab program for the forward Euler method will be provided on the class web site.
   How large can you make Δt for each method? What happens when Δt is too large? Compare your numerical result for $V(t = 20)$ with the analytical result. Measure for each solution method the error $|V(t = 20) - V_p(t = 20)|$ as a function of Δt, changing Δt by factors of 2 over at least a range of 16. Does the error decrease at the rate expected for these numerical methods?

3. **Hodgkin-Huxley Neuron**

Investigate the Hodgkin-Huxley equations numerically using either hhsim or the matlab code on the web site. Use the parameters as given in Dayan&Abbott on p.173 after eq.(5.25) and in eqs.(5.22,5.24). Use as initial conditions the rest state.

(a) Refractory period: give two current injections with magnitude $I_{1,2} = 3\mu A/cm^2$ and duration of $\Delta t = 5ms$ at $t_1 = 10ms$ and at a time t_2. Decrease t_2 from $t_2 = 30ms$ to $t_2 = 15ms$. How many spikes do the two current injections trigger? Why? For $t_2 = 25ms$ and $t_2 = 20ms$ measure (roughly) the minimal stimulation I_2 that is necessary to excite a second action potential.

(b) Turn off the K^+-channel (set $g_K = 0$). Without any current injection you will observe a strong depolarization of the cell. Why? Why does the voltage decay after the initial maximum?

(c) Now turn off the Na^+-channel with the K^+-channel turned on. Give a stimulus $I = 100\mu A/cm^2$ with a duration of $\Delta t = 10ms$. Why does the voltage overshoot both at the stimulus onset and its offset.

(d) For a steady current injection I_0 with duration $\Delta t = 200ms$ measure the frequency of spiking. A good measure for the period is the time between zero-crossings of the voltage. Plot the frequency as a function of I_0 in the range $6\mu A/cm^2 \leq I_0 \leq 8\mu A/cm^2$ (in steps of $0.1\mu A/cm^2$ near the lower end of firing range. What is the minimal stimulation to obtain steady firing? What happens for lower values of I_0?