C-21-1 Models in Applied Mathematics
Fall 1995
Prof. Hermann Riecke

Problem Set 5 November 13

Due November 30

This time I have picked problems from Lin&Segel:

1. Suppose that the spatial coordinates x_i of a particle at time t are given by

$$x_1 = R_1 + t, \qquad x_2 = R_2\, e^t, \qquad x_3 = R_3 + tR_1.$$

Find the velocity vector $\mathbf{v}(\mathbf{r}, t)$, i.e., transform from the material to the spatial description. (Problem 1 in Sec.13.2; solution is given in the back of the book)

2. In a two-dimensional *unsteady* flow the velocity components are given by

$$v_x = t, \qquad v_y = x + 1.$$

a) Find the streamline passing through (x_0, y_0).
b) Find the particle path that commences at (x_0, y_0).
c) Plot the streamline and the particle path from a) and b) for a suitable range of t.
(Problem 4 in Sec.13.2; note c) is additional; solution for a) and b) is given in the back of the book)

3. In the Eulerian description, a two-dimensional flow is given by the steady velocity field

$$v_x = 2x + 3y, \qquad v_y = x - y.$$

a) Find the corresponding Lagrangian description.
b) Plot the path of particles that pass through the point $(x = 2, y = 0)$ sometime during the motion. Note that all such particles have the same path, since the motion is steady.
c) Find the acceleration of the particle that is instantaneously located at (3,4). Comment on the fact that the answer is nonzero, although the flow is steady.
(Problem 4 in Sec.13.3)