C-21-2 Models in Applied Mathematics
Winter 1997
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Problem Set 5 January 14, 1997

Due January 23

This time I have picked problems from Lin&Segel:

1. Suppose that the spatial coordinates x_i of a particle at time t are given by

$$x_1 = R_1 + t, \qquad x_2 = R_2\, e^t, \qquad x_3 = R_3 + t R_1.$$

   Find the velocity vector $\mathbf{v}(\mathbf{r}, t)$, i.e., transform from the material to the spatial description. (Problem 1 in Sec.13.2; solution is given in the back of the book)

2. In a two-dimensional *unsteady* flow the velocity components are given by

$$v_x = t, \qquad v_y = x + 1.$$

   a) Find the streamline passing through (x_0, y_0).
   b) Find the particle path that commences at (x_0, y_0).
   c) Using your favorite computer program, plot the streamline and the particle path from a) and b) for a suitable range of $t_0 \leq t \leq t_1$ and suitable values of (x_0, y_0). Choose the values t_0, t_1, and (x_0, y_0) such that the connection between streamline and particle path becomes apparent, i.e. at least at one time the particle should be on the streamline. Comment on the result: does the particle follow the streamline?
   (Problem 4 in Sec.13.2; note c) is additional; solution for a) and b) is given in the back of the book)

3. In the Eulerian description, a two-dimensional flow is given by the steady velocity field

$$v_x = 2x + 3y, \qquad v_y = x - y.$$

   a) Find the corresponding Lagrangian description.
   b) Plot the path of particles that pass through the point $(x = 2, y = 0)$ sometime during the motion. Note that all such particles have the same path, since the motion is steady.
   c) Find the acceleration of the particle that is instantaneously located at (3,4). Comment on the fact that the answer is nonzero, although the flow is steady.
   (Problem 4 in Sec.13.3)