**C-21-2 Models in Applied Mathematics**
Winter 1997
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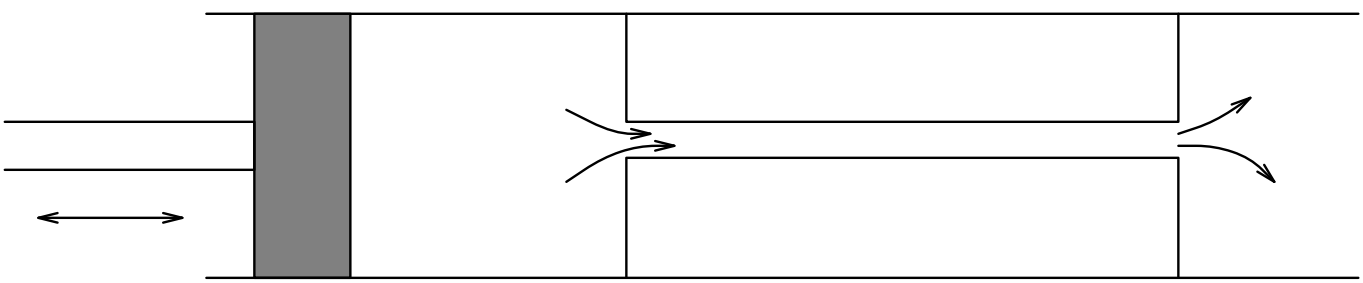
**Problem Set 4** February 18, 1997

Due March 4, 1997

1. **Shock Absorber and Stokes Layers**

Consider an oil-shock absorber in a car bumping along the Kennedy before its renovation. To get an impression of the flow of the oil in the pipes of such a shock-absorber exposed to a bumpy road, consider as a very simple model the flow of a fluid through a channel and assume the flow is driven by a time-periodic pressure drop along the channel.
a) What is the simplest set-up that gives still insight into the relevant features of that flow? Can the fluid taken to be inviscid, irrotational, parallel or incompressible? What type of boundary conditions should be taken? What geometry of the channel and time-periodic function simplifies the calculation without loosing too much relevant information?
b) Describe qualitatively how the velocity profile will look like. Will it exhibit any spatial symmetries? Use dimensional analysis to give characteristic scales for the flow (i.e. a decay length, a typical velocity).
c) Calculate the flow field that is obtained after transients have decayed, i.e. the time-periodic flow field that is obtained as a steady state. You will obtain a linear partial differential equation with time-periodic coefficients. It is most convenient to make a complex ansatz for the solution (rather than a real one). In the final step one would have to calculate the real part of the complex solution obtained. This may involve a lot of algebra. Don't do that. Just confirm your expectations (from b)) for the decay length and the scale of the velocity.
d) Plot the solution $u(z,t)$ as a three-dimensional plot using a program like Maple. In case you are not familiar with Maple: a core program to plot $f(x,t) = A * sin(x) * sin(\omega t)$ could look like
A:=2;
omega:=1; f:=(x,t)->A*sin(x)*sin(omega*t);
plot3d(f(x,t),x=0..2*Pi/omega,t=0..4*Pi,grid=[30,30],axes='framed',title='test plot');
Note the difference in the apostrophes around framed and test plot.