D 46-1
Numerical Solution of Partial Differential Equations
Fall 1996
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Problem Set 2
Due October 10 in class.

2. ODE: Nonlinear Duffing Oscillator using MATLAB

Solve the Duffing oscillator

$$\frac{d^2u}{dt^2} = -\partial_u V(u), \text{ with } V(u) = -\frac{1}{2}u^2 + \frac{1}{4}u^4. \tag{1}$$

using MATLAB. Use again a fourth-order Runge-Kutta scheme[1]. Use the MATLAB program which you get by e-mail as a template. It solves the equation with forward Euler and provides you with a control window and two display windows. In one of the display windows the solution $u = u(t)$ is shown as a function of time, in the other it is displayed in phase space, i.e. $\partial_t u$ is plotted as a function of u. The latter is very useful to understand the behavior of the oscillator. The program also writes the solution to a file $duffing.dat$.

1. Go through all the subroutines ("M-files") of the provided template and make sure you understand what they are doing.

2. Compare the accuracy of your MATLAB program with that of your FORTRAN/C program. To do that calculate the solution with initial conditions $u(t = 0) = 1.414214$ and $\partial_t u(t = 0) = 0$ up to time $t = 100$ and a dt which is required for graphical resolution.

3. Print a phase-space plot that demonstrates the effect of the change in the total energy, which is due to the numerical error, on the solution. Use $dt = 0.05$ and run up to $t = 100$ with the same initial condition as 1).

Suggestion:
For the person interested in complex dynamics it is worthwhile to study what happens if the Duffing oscillator is forced periodically, i.e.

$$\frac{d^2u}{dt^2} = -\partial_u V(u) + \alpha \cos \omega t, \text{ with } V(u) = -\frac{1}{2}u^2 + \frac{1}{4}u^4. \tag{2}$$

For initial conditions close to the unstable fixed point $u = 0$ even small time-dependent perturbations have a strong effect since they can destroy the homoclinic orbit connecting $u = 0$ with itself.

[1] You can either write your own routine or use that provided in MATLAB. In either case you want to plot the solution in the two display windows.