D-46-1
Numerical Solution of Partial Differential Equations
Fall 1996
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Problem Set 3

This problem set is due 10/22/96 in class.

3. Different Approximations for Spatial Derivatives

1. Derive the 6th order central difference approximation of d^2u/dx^2, i.e. the error will be of $O(\Delta x^6)$. (Use either Taylor expansion of $u(x+h)$ or a Fourier analysis). Find the Fourier representation of the operator obtained in this approximation and the leading-order term in the relative error.

2. Derive the second-order approximation of du/dx using forward differences, i.e. involving only points to the right of the point at which the derivative is to be approximated. Do a Fourier analysis of this approximation and find again the leading-order term in the relative error (in real and in imaginary part). Discuss the effect of the two terms if this scheme is used in the PDE $\partial_t u = \partial_t u$.

3. Use the result of problem 2 to find the second-order backward-difference approximation of du/dx. Do not derive it from scratch!

4. Show that every central-difference approximation for du/dx vanishes at $kh = \pi$.