D 46-1
Numerical Solution of Partial Differential Equations
Fall 1996
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Problem Set 4

3. Neumann Analysis of Finite-Difference Schemes

1. Perform a Neumann analysis for the (2,4)-leap-frog scheme of the one-way wave equation

$$\partial_t u = c\partial_x u, \tag{1}$$

   i.e. use a fourth-order central-difference approximation for the spatial derivative and a second-order central-difference approximation for the temporal derivative. What is the stability limit of this scheme?

2. Perform a Neumann analysis for the (2,2)-MacCormack scheme of the two-way wave equation

$$\partial_t u = \frac{1}{\rho}\partial_x v, \qquad \partial_t v = \rho c^2 \partial_x u. \tag{2}$$

   What is the stability limit of this scheme? Note that the easiest way to treat coupled PDE is to consider the vector $\mathbf{w} = (u, v)$ and write the equations as a matrix differential equation. How many solutions z do you obtain? How are they related? How many different values of $|z|$ are there?

This problem set is due 10/29/96 in class.