D-46-2
Numerical Solution of Partial Differential Equations
Winter 1997
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Problem Set 2, February 25

due March 4, 1997

1. Chebyshev Approximation for Derivatives

Write a program which calculates the derivative of the functions $f_i(x)$ below using the pseudospectral Chebyshev approximation. Do this with the matrix-multiply technique using the Chebyshev derivative matrix D_{ij}. Measure the maximal error of the approximation, i.e. $max\{\partial_x I_N(f(x_i)) - \partial_x f(x_i)\}$, for an increasing number of grid points $N = 2^n$ until the accuracy is only limited by round-off errors. Show the error in a linear-log plot and compare the convergence rates for the various cases.
a) Use $f_1(x) = \sin(20x)$.
b) Use $f_2(x) = \tanh((x - x_0)/\xi)$ with
i) $x_0 = 0$, $\xi = 1$, ii) $x_0 = 0$, $\xi = 0.1$ and iii) $x_0 = -0.9$, $\xi = 0.1$.
Interpret the difference in the convergence properties of these three cases.

A program to calculate D_{ij} comes by e-mail.